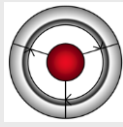


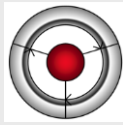
A Two-Stage Approach Corrects a Fundamental Bias in Secured Loan LGD Modeling

Dr. Hendrik Wagner, RiskParameters.EU
Edinburgh, CSCCXIII, August 28-30 2013



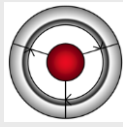
Problem

- Estimating the recovery rate ($= 1 - \text{haircut}$) that is going to be experienced when selling physical collateral is the central component of LGD models in secured lending
- Recovery rate is typically estimated as a simple mean of (sales proceeds / collateral value) across sales
- However, this is overly optimistic, because the revenue that the bank receives from each sale is capped by the exposure.



Typical Secured Loan LGD Model

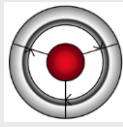
- Predicted LGD = $1 - \text{Predicted Revenue} / \text{Exposure At Default}$
 - Predicted Revenue = $\min(\text{Mean Recovery Rate} * \text{Collateral Value}, \text{Exposure At Default}) = \min(\text{Mean Recovery Rate}, \text{LTV}) * \text{Collateral Value} = \text{Effective Recovery Rate} * \text{Collateral Value}$
 - $\text{LTV} = \text{Exposure at Default} * \text{Collateral Value}$
 - Mean Recovery Rate = mean of historically realized ratios of (Sales Proceeds / Collateral Value)
- Note that the Mean Recovery Rate is typically calculated on the basis of sales proceeds alone, not accounting for the fact that the bank actually has access to the whole sales proceeds only if the sales proceed do not exceed the exposure



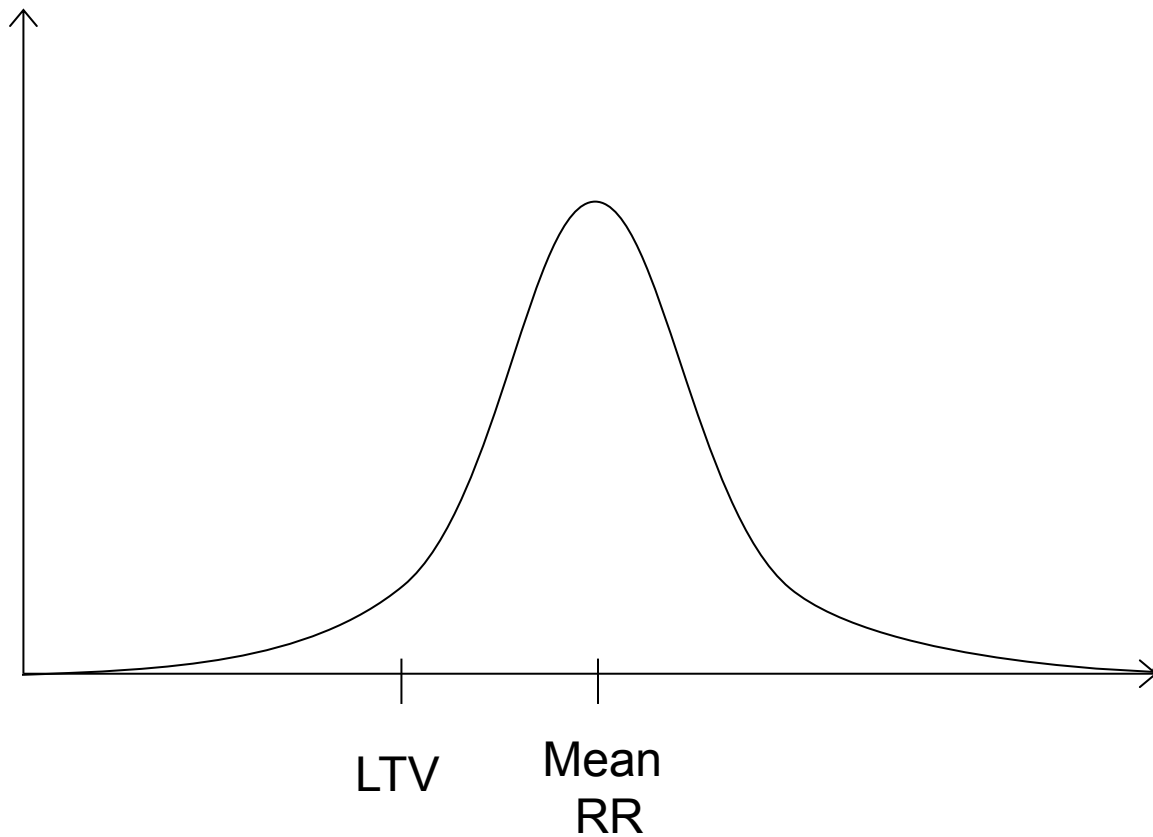
Two Situations Cause Bias (1/2)

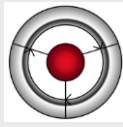
1) LTV of a case is smaller than Mean RR

- Predicted revenue is $LTV * Collateral Value = Exposure At Default$
- Predicted LGD = 0
- However, in some cases actual RR may be so much lower than the Mean RR that it is even lower than LTV, resulting in an unpredicted loss
- Therefore, on the whole the predictions are too optimistic

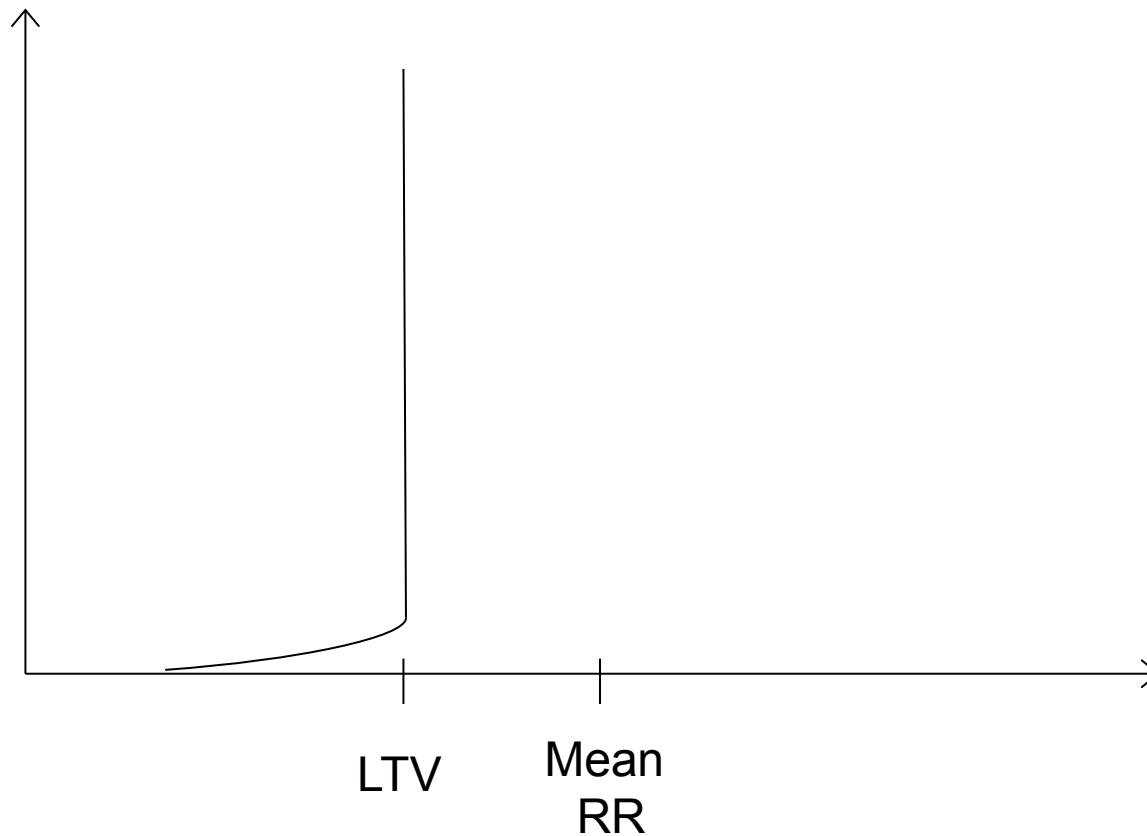


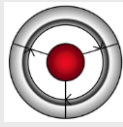
Distribution of RR



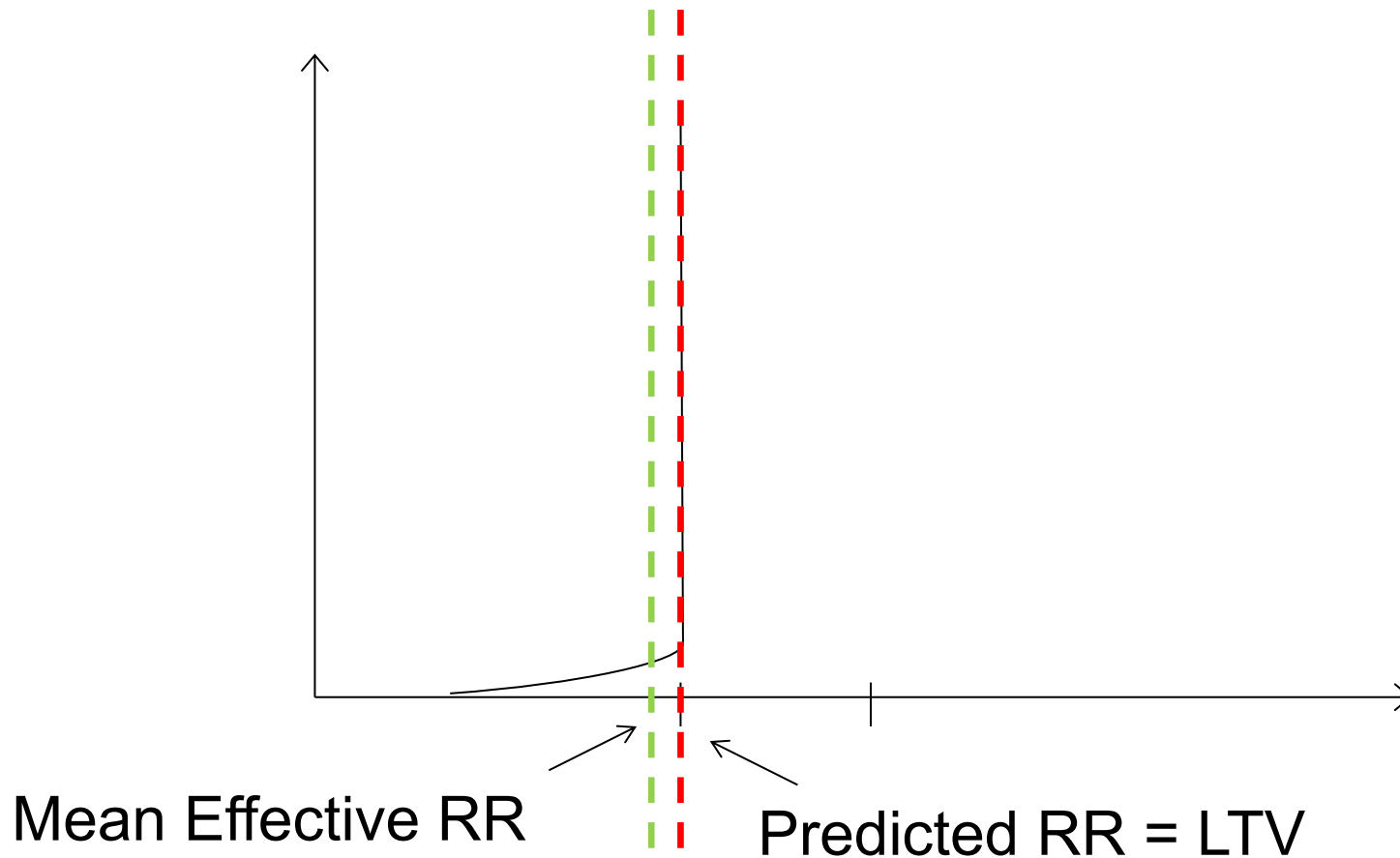


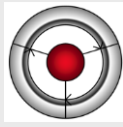
Distribution of Effective RR





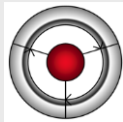
Predicted and Mean Effective RR are Different





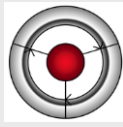
Example 1: LTV of a case is smaller than Mean RR

- Portfolio of ten identical cases in terms of Exposure at Default (EAD) and Collateral Value, with an LTV of 20%
- The Actual Recovery Rate (sales proceeds / collateral value) is assumed to vary around a Mean RR of 80%
- The Predicted Revenue is always equal to the EAD, because the Mean RR is always greater than the LTV
- However, in one of the ten cases, the Actual RR is smaller than the LTV, resulting in lower than predicted revenue



Example 1: LTV of a case is smaller than Mean RR

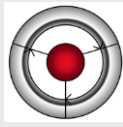
Case Nr	EAD	Value	LTV	Actual RR	Mean RR	Prediction Revenue	Prediction LGD	Actual Revenue	Actual LGD
1	20	100	0.2	0.1	0.8	20	0.00	10	0.50
2	20	100	0.2	0.9	0.8	20	0.00	20	0.00
3	20	100	0.2	0.9	0.8	20	0.00	20	0.00
4	20	100	0.2	0.7	0.8	20	0.00	20	0.00
5	20	100	0.2	1	0.8	20	0.00	20	0.00
6	20	100	0.2	1	0.8	20	0.00	20	0.00
7	20	100	0.2	1	0.8	20	0.00	20	0.00
8	20	100	0.2	1	0.8	20	0.00	20	0.00
9	20	100	0.2	0.7	0.8	20	0.00	20	0.00
10	20	100	0.2	0.7	0.8	20	0.00	20	0.00
average	20	100	0.2	0.8	0.8	20	0.00	19	0.05



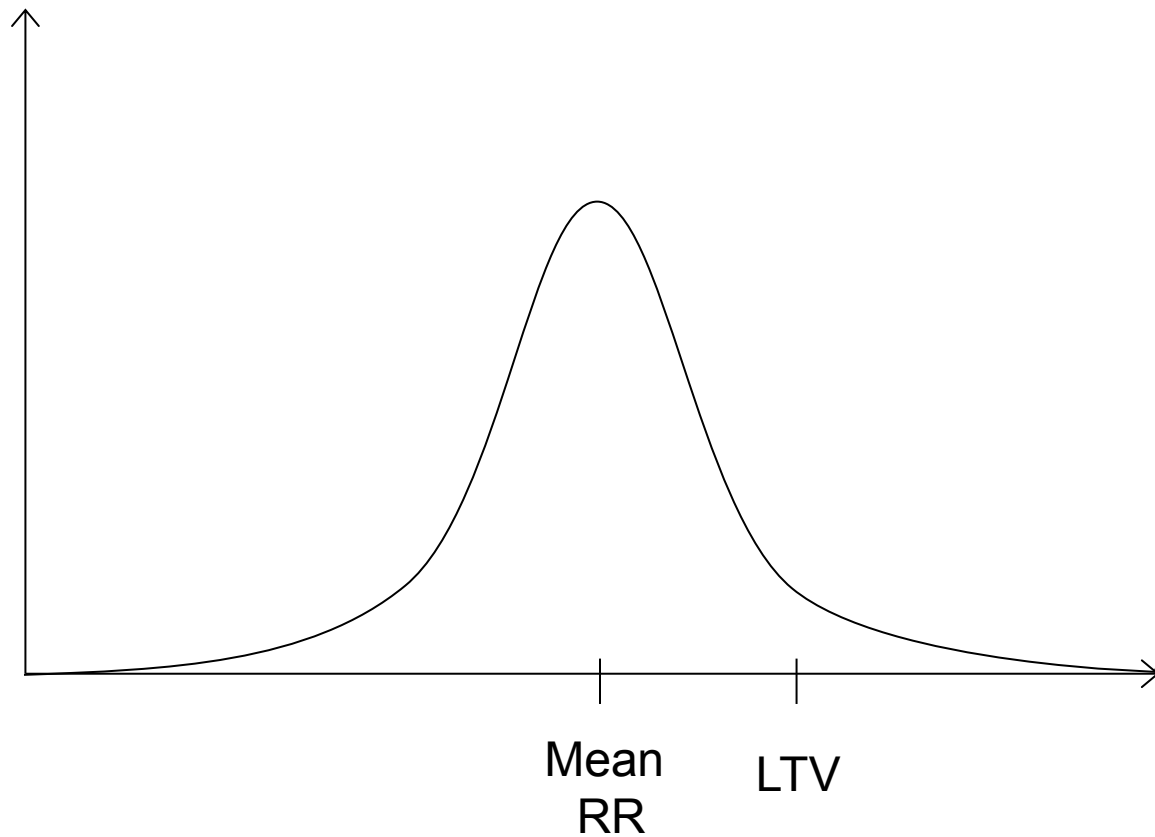
Two Situations Cause Bias (2/2)

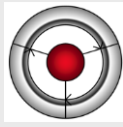
2) LTV of a case is greater than Mean RR

- Predicted revenue is $\text{Mean RR} \times \text{Collateral Value} < \text{Exposure}$
- Predicted LGD > 0
- However, in some cases actual RR will even exceed LTV. The revenue will then be capped to the exposure and the effective actual RR will equal LTV. While this is still greater than the Mean RR, it is not anymore sufficiently great to offset the downward deviations on the other end of the RR distribution
- Therefore, because the capping is not part of the Mean RR calculations, on the whole the predictions are too optimistic

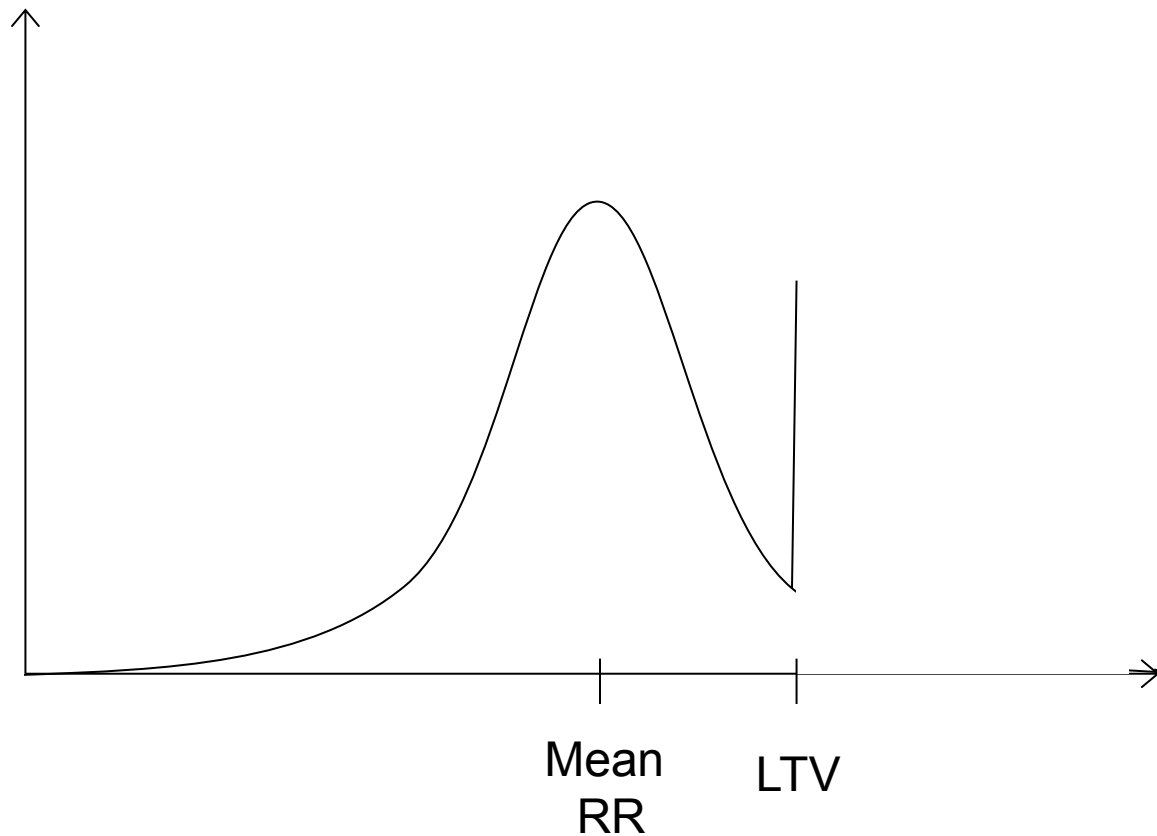


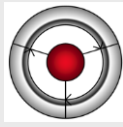
Distribution of RR



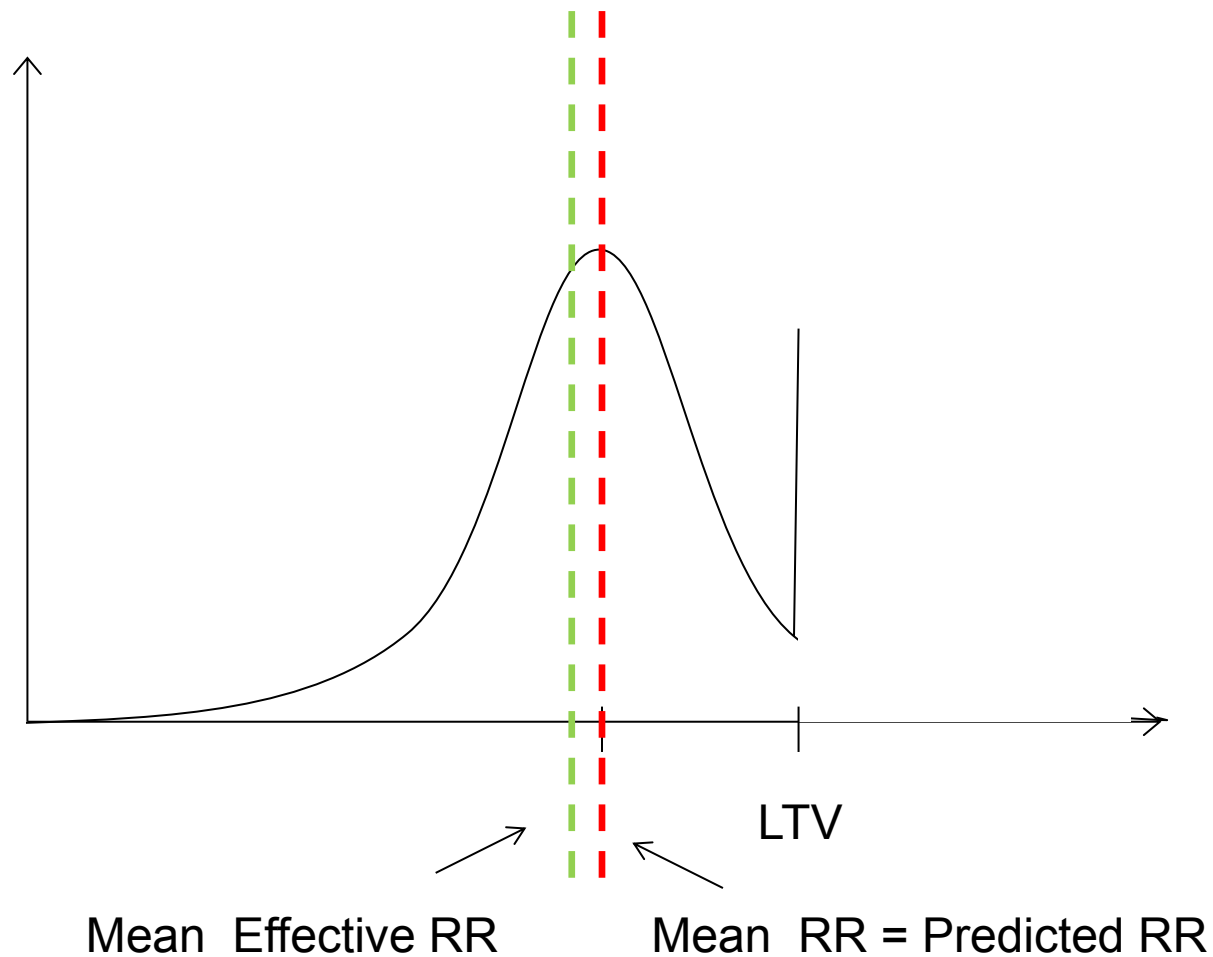


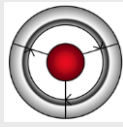
Distribution of Effective RR





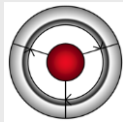
Predicted and Mean Effective RR are Different





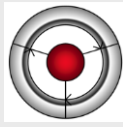
Example 2: LTV of a case is greater than Mean RR

- Portfolio of ten identical cases in terms of EAD and Collateral Value, with an LTV of 90%
- The Actual Recovery Rate (sales proceeds / collateral value) is assumed to vary around a Mean RR of 80%
- The Predicted Revenue is always smaller than the EAD, because the Mean RR is always smaller than the LTV
- However, in four of the ten cases, the Actual RR is greater than the LTV, resulting in capped revenue that causes the average revenues to be smaller than predicted



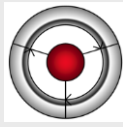
Example 2: LTV of a case is greater than Mean RR

Case Nr	EAD	Value	LTV	Actual RR	Mean RR	Prediction Revenue	Prediction LGD	Actual Revenue	Actual LGD
1	90	100	0.9	0.1	0.8	80	0.11	10	0.89
2	90	100	0.9	0.9	0.8	80	0.11	90	0.00
3	90	100	0.9	0.9	0.8	80	0.11	90	0.00
4	90	100	0.9	0.7	0.8	80	0.11	70	0.22
5	90	100	0.9	1	0.8	80	0.11	90	0.00
6	90	100	0.9	1	0.8	80	0.11	90	0.00
7	90	100	0.9	1	0.8	80	0.11	90	0.00
8	90	100	0.9	1	0.8	80	0.11	90	0.00
9	90	100	0.9	0.7	0.8	80	0.11	70	0.22
10	90	100	0.9	0.7	0.8	80	0.11	70	0.22
average	90	100	0.9	0.8	0.8	80	0.11	76	0.16

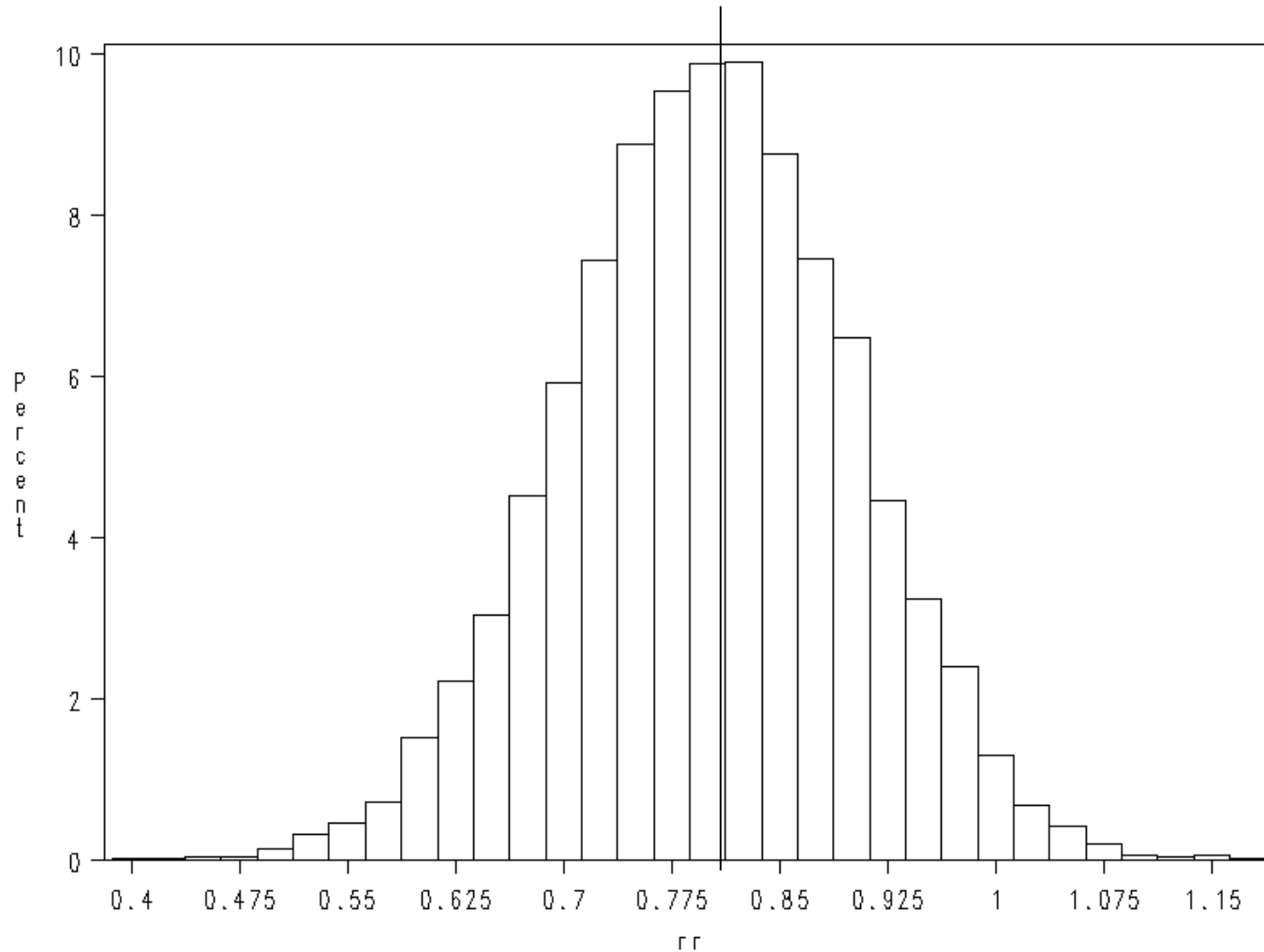


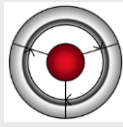
Simulations

- In real life, one deals with more or less overlapping distributions of LTV and RR
- The simulations are based on independent normal distributions of LTV and RR
 - (!?)

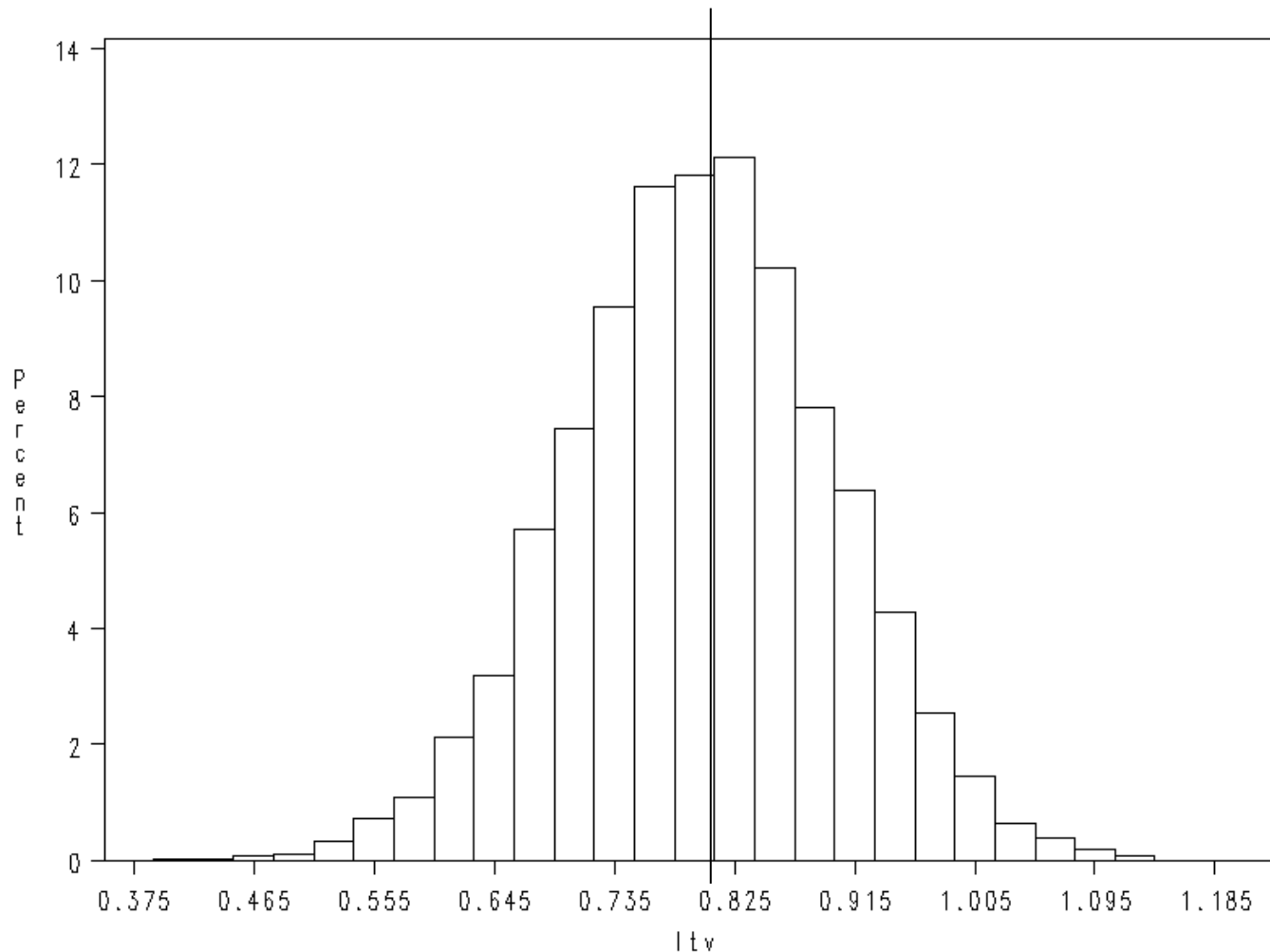


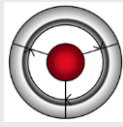
Distribution of RR (mean=0.8 std=0.1)



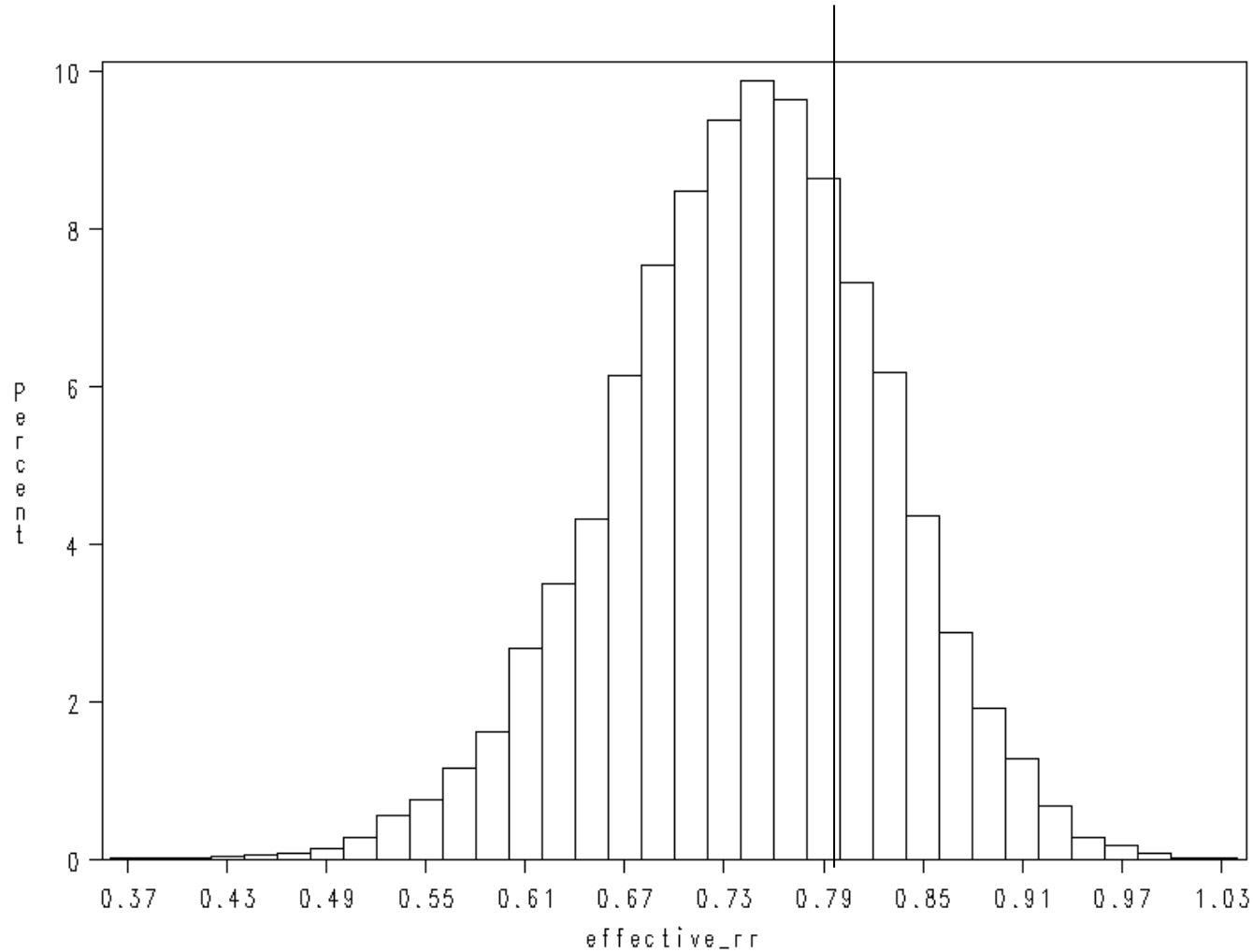


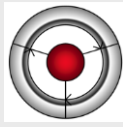
Distribution of LTV (mean=0.8 std=0.1)



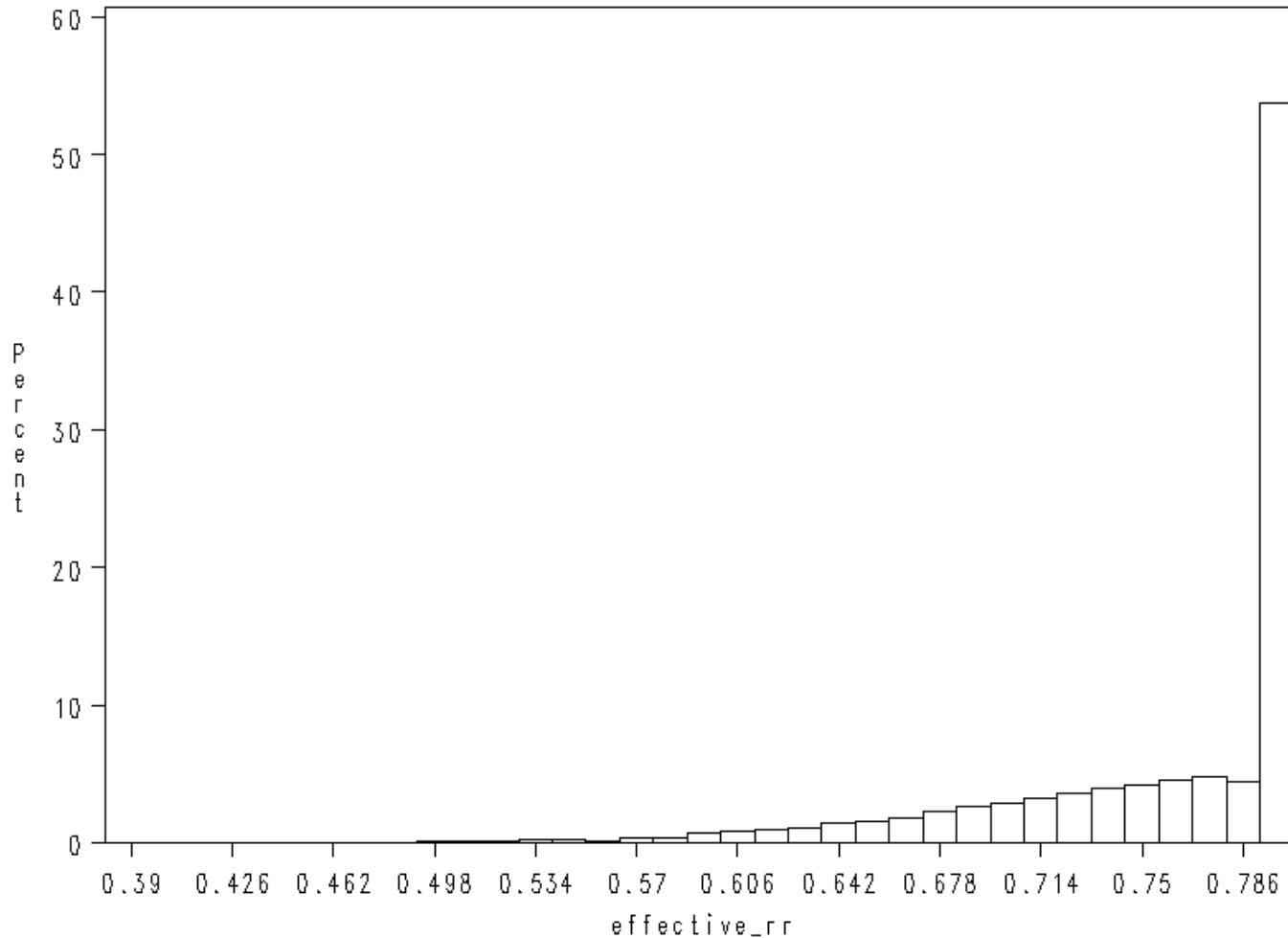


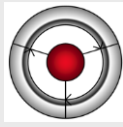
Distribution of Effective RR



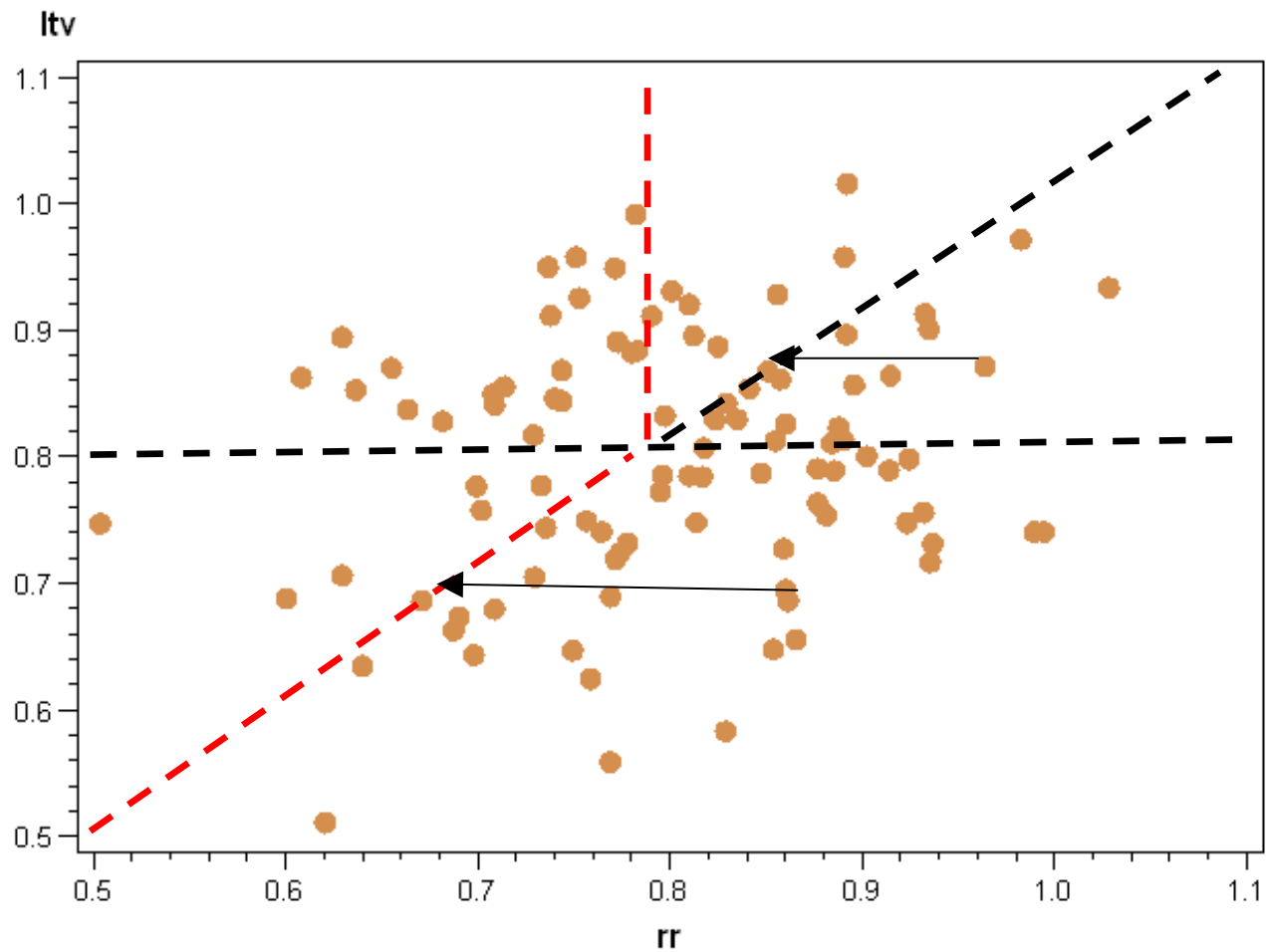


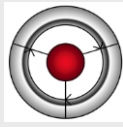
Distribution of Effective RR for LTV=0.8



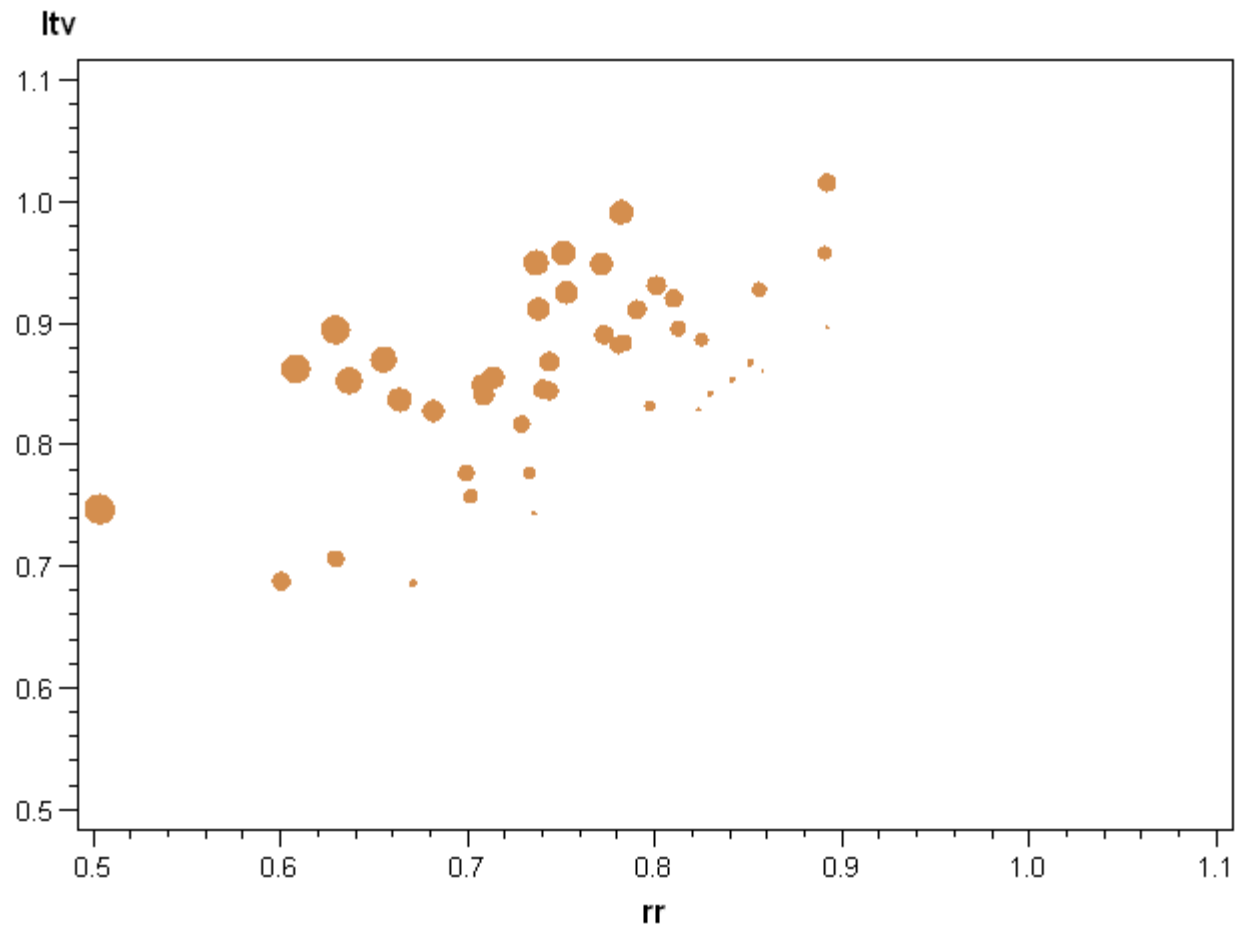


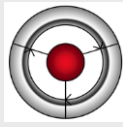
Scatterplot LTV vs RR



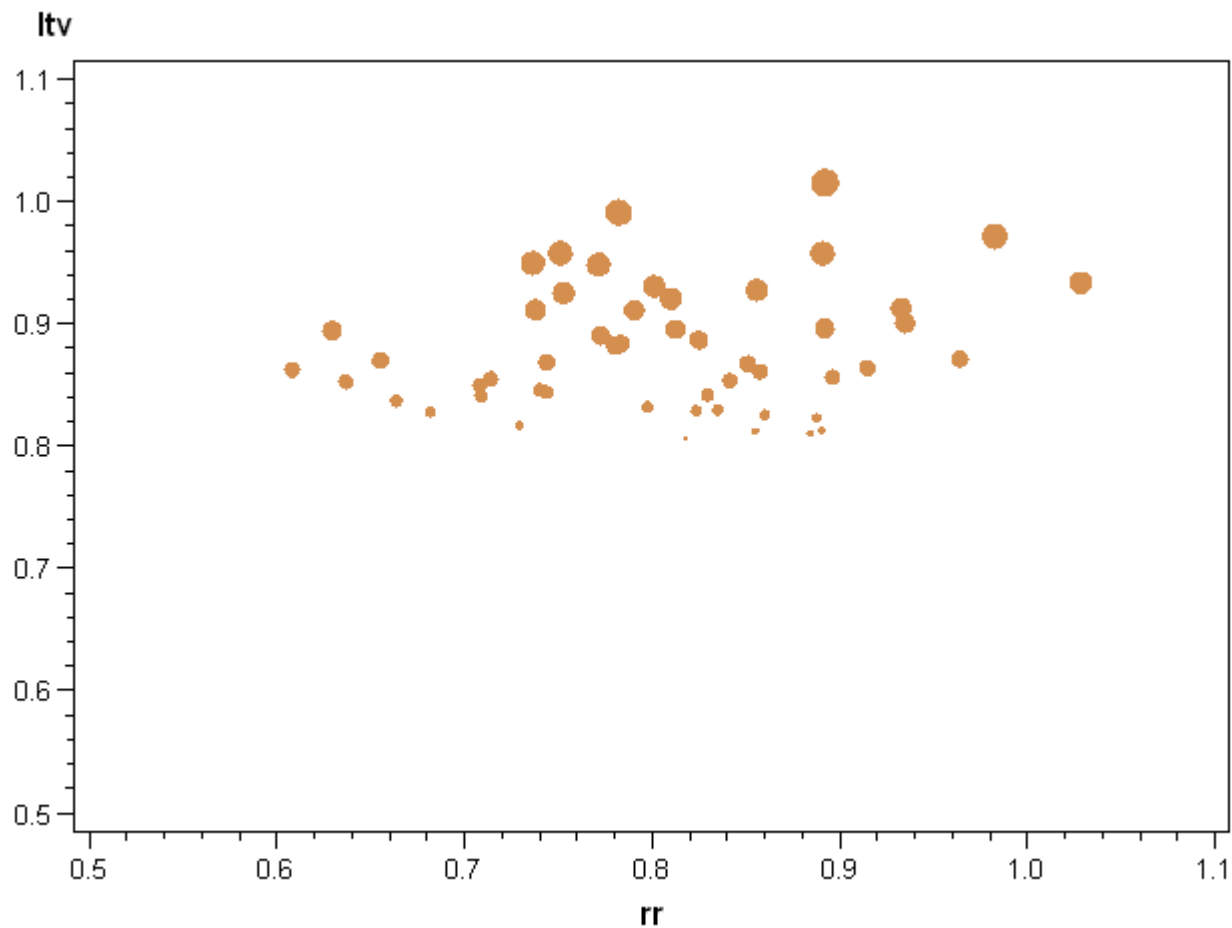


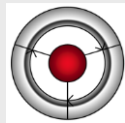
$$\text{LGD} = f(\text{RR}, \text{LTV})$$





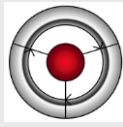
Predicted LGD = f(RR,LTV)



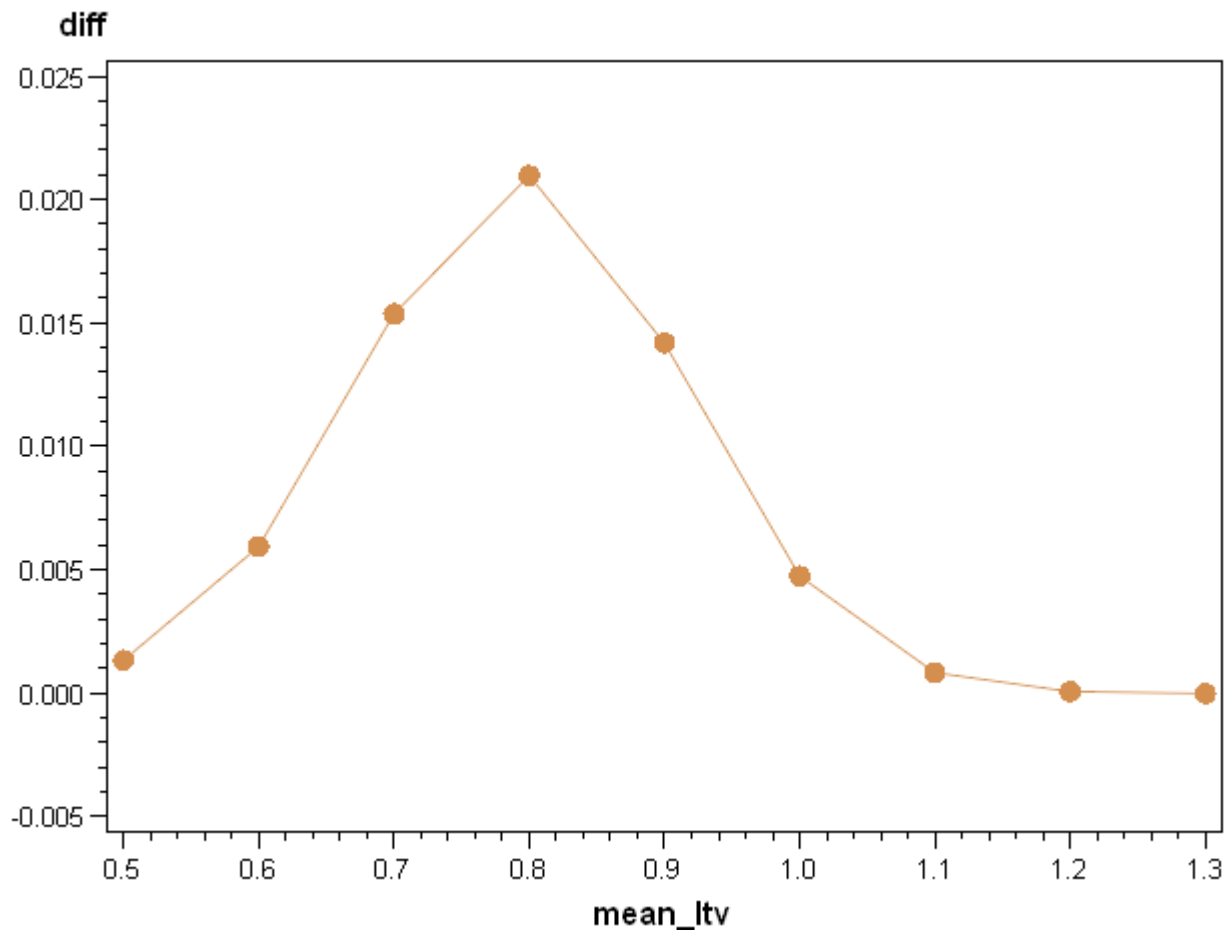


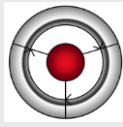
Overall Means

Variable	Mean	N	Std Dev
ltv	0.8000000	10000	0.1000000
rr	0.8000000	10000	0.1000000
lgd	0.0641159	10000	0.0910706
predicted_lgd	0.0431487	10000	0.0600829

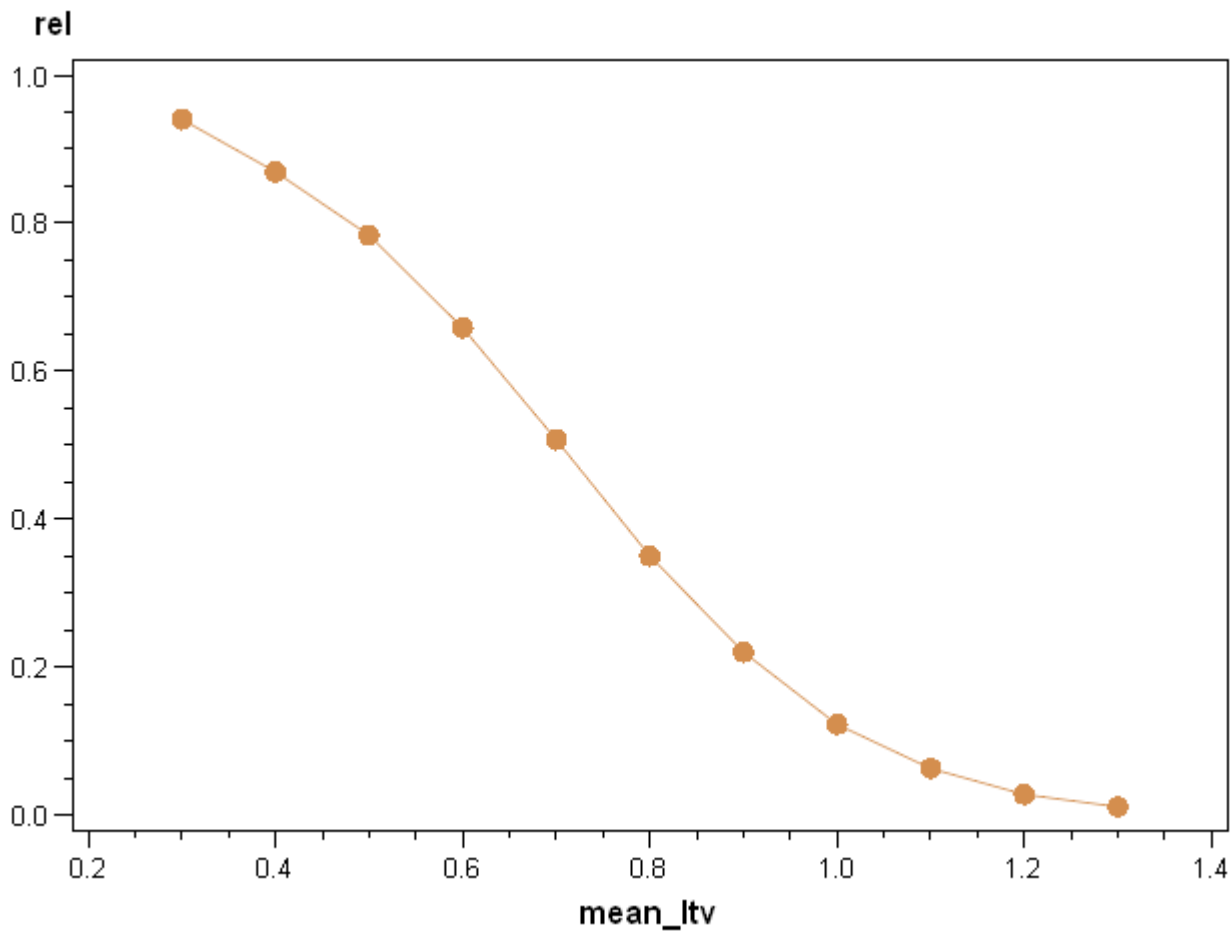


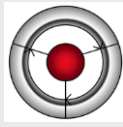
Mean LGD – Mean Predicted LGD = f(Mean LTV)
for Mean RR=0.8





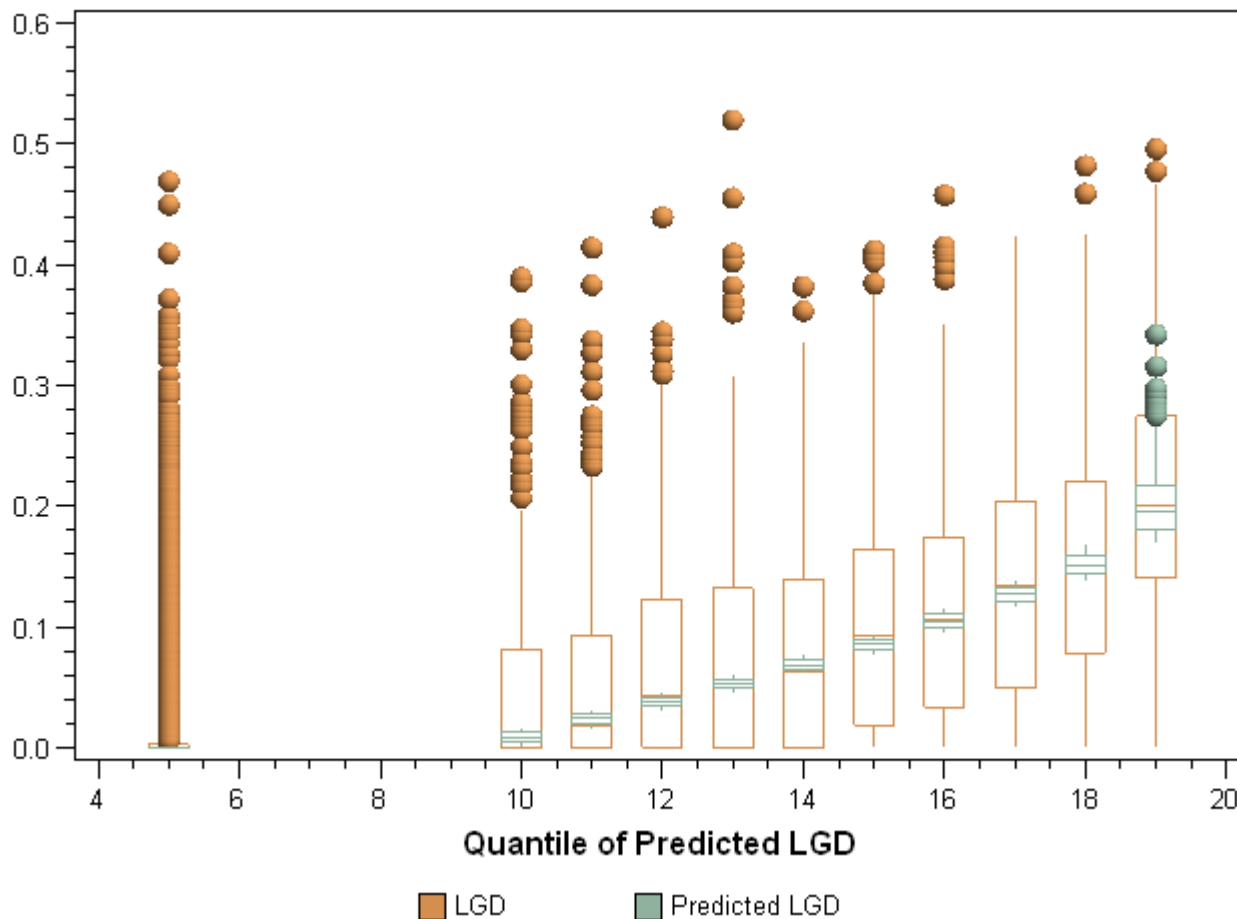
$(\text{Mean LGD} - \text{Mean Predicted LGD}) / \text{Mean LGD} =$
 $f(\text{Mean LTV})$ for Mean RR=0.8

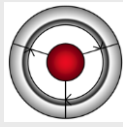




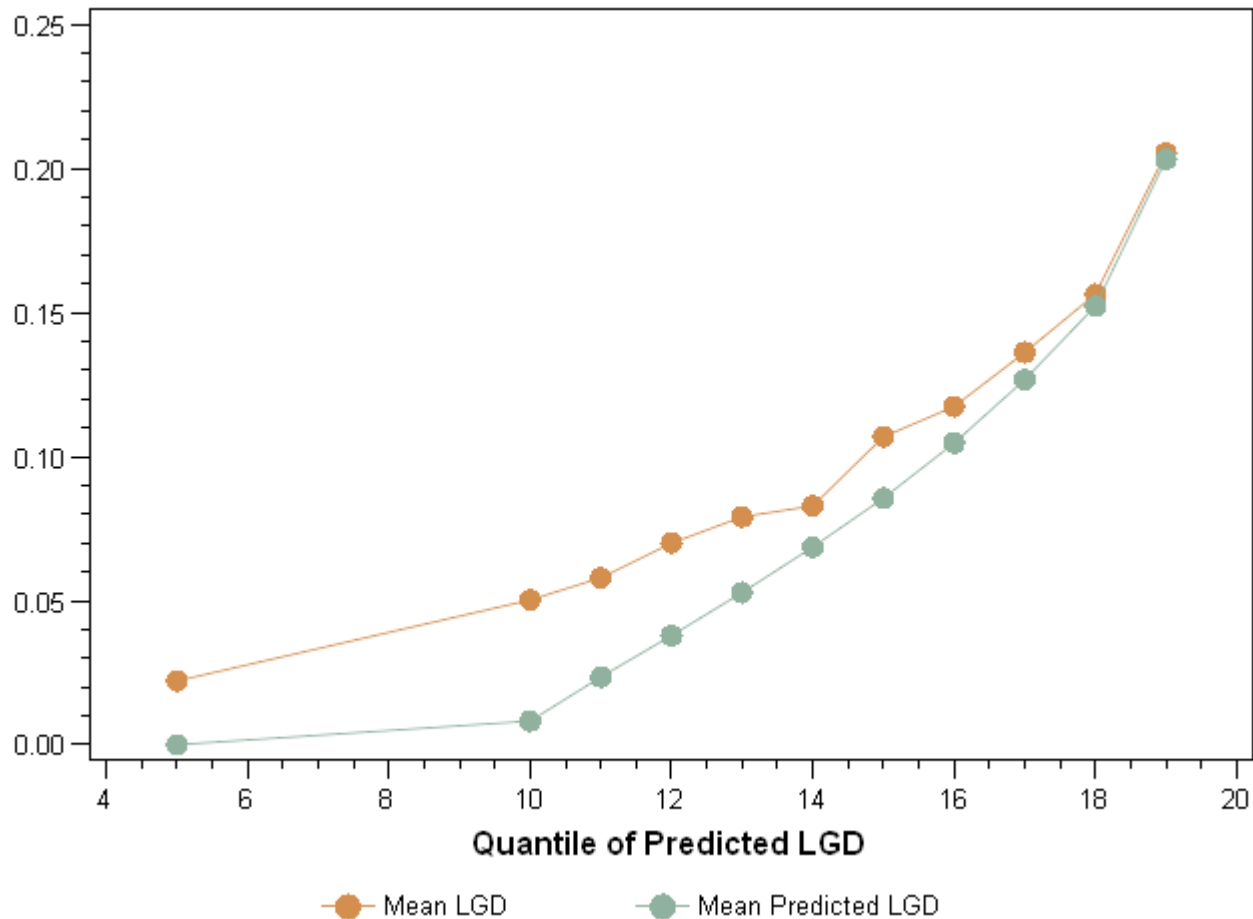
LGD and Predicted LGD - Distributions by Quantile

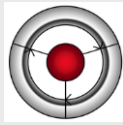
Mean RR=0.8 and Mean LTV=0.8





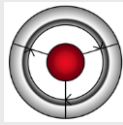
LGD and Predicted LGD - Means by Quantile Mean RR=0.8 and Mean LTV=0.8





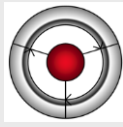
Solution (1/3)

- A two-stage approach can be applied
- For every LTV
 - Estimate the probability that the LTV is greater than the RR (resulting in non-zero loss)
 - Then estimate the RR under this condition
- An adjusted LGD estimate can then be written as a function of LTV



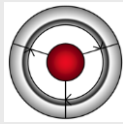
Solution (2 / 3)

- $LGD = 1 - \text{Revenue} / \text{EAD}$
 - $\text{Revenue} = \min(\text{RR} * \text{Value}, \text{EAD}) = \min(\text{RR}, \text{LTV}) * \text{Value}$
 - $\text{LTV} = \text{EAD} / \text{Value}$
- $\text{LTV} > \text{RR} \quad \Leftrightarrow \quad 1 > \text{LGD} > 0$
- $\text{LTV} \leq \text{RR} \quad \Leftrightarrow \quad \text{LGD} = 0$
- $$\begin{aligned} E(\text{LGD} \mid \text{LTV}) &= P(\text{LTV} > \text{RR}) * E(\text{LGD} \mid \text{LTV} > \text{RR}) + P(\text{LTV} \leq \text{RR}) * 0 \\ &= \mathbf{P(\text{LTV} > \text{RR}) * (1 - \text{Value} * \mathbf{E(\text{RR} \mid \text{LTV} > \text{RR})} / \text{EAD})} \end{aligned}$$



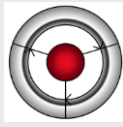
Solution (3/3)

- Estimate $P(\text{LTV} > \text{RR})$ by means of a logistic regression
 - $\ln(P/(1-P)) = \alpha_p + \beta_p * \text{LTV}$
 - On all cases
- Estimate $E(\text{RR} \mid \text{LTV} > \text{RR})$ by means of a linear regression
 - $E = \alpha_e + \beta_e * \text{LTV}$
 - Only on cases where $\text{LTV} > \text{RR}$

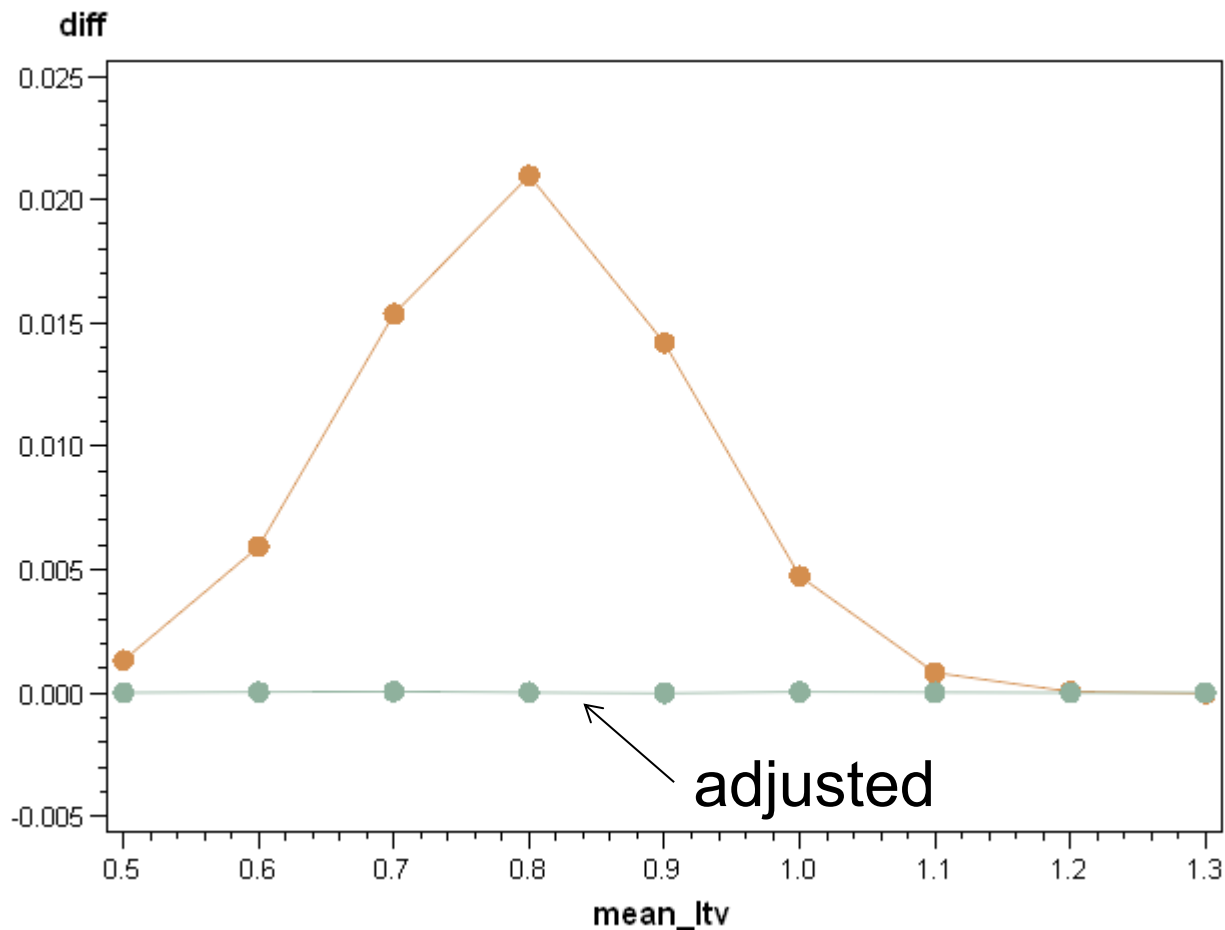


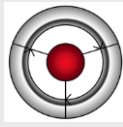
Overall Means

Variable	Mean	N	Std Dev
ltv	0.8000000	10000	0.1000000
rr	0.8000000	10000	0.1000000
lgd	0.0641159	10000	0.0910706
predicted_lgd	0.0431487	10000	0.0600829
adj_lgd	0.0641020	10000	0.0542779



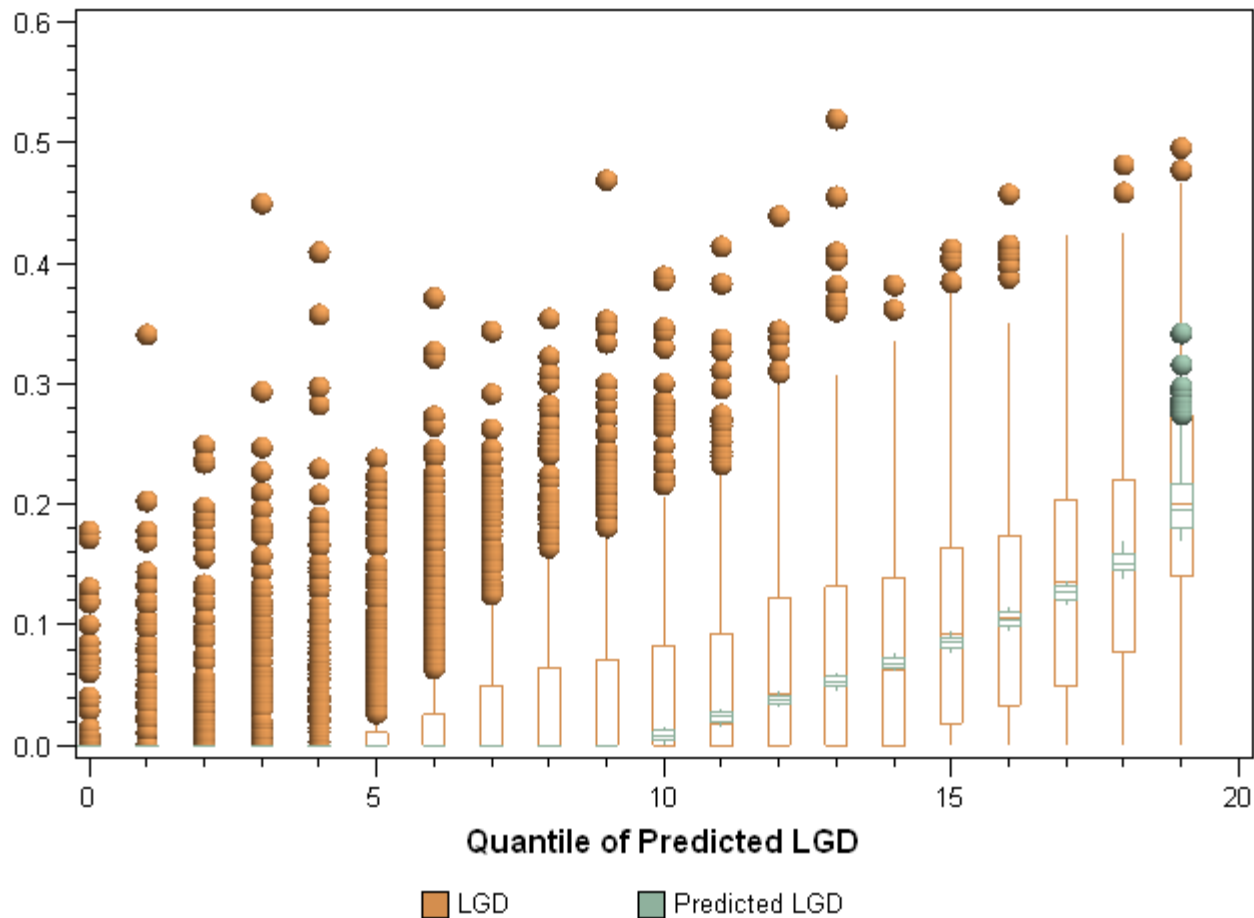
Mean LGD – Mean Predicted LGD = f(Mean LTV)
for Mean RR=0.8

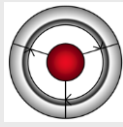




LGD and Adjusted Predicted LGD - Distributions by Quantile

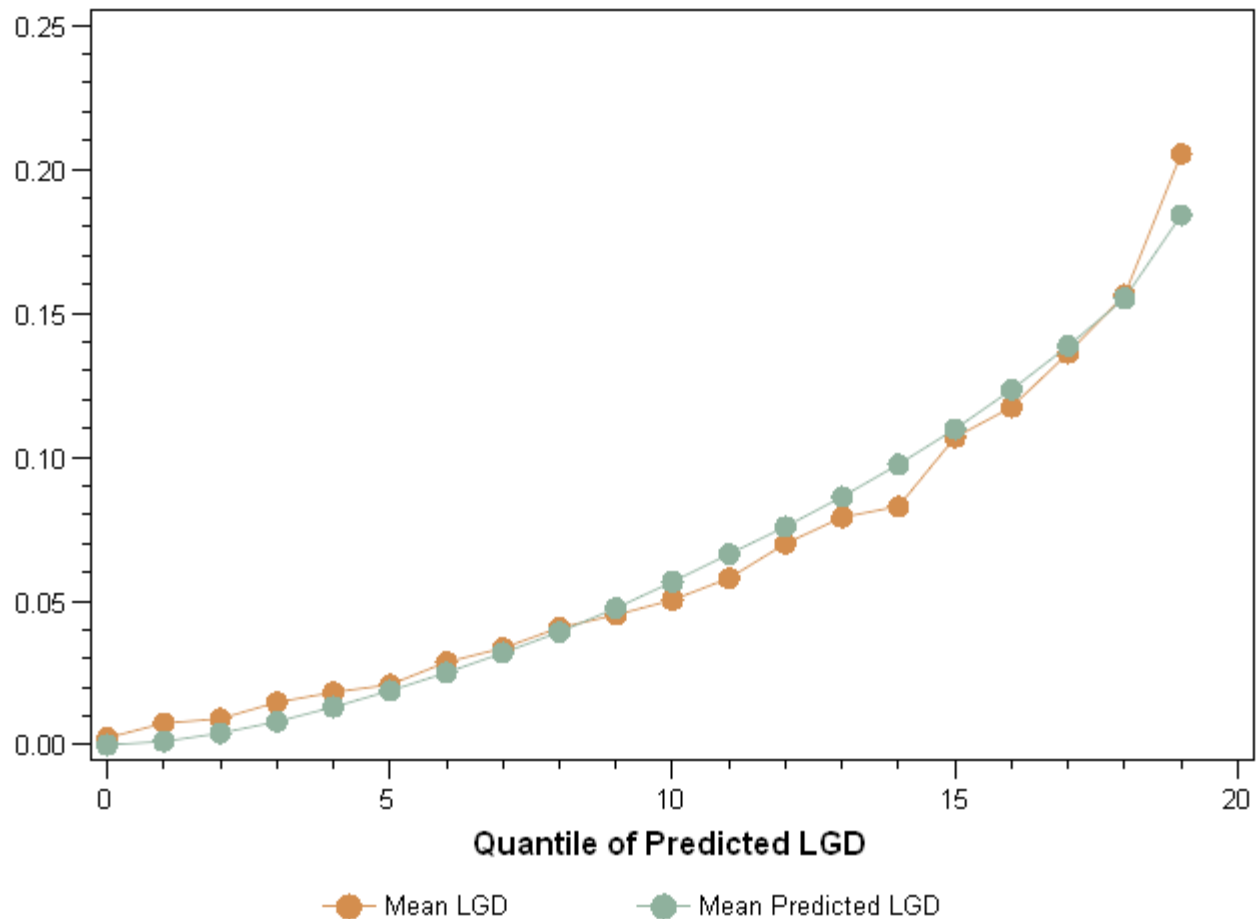
Mean RR=0.8 and Mean LTV=0.8

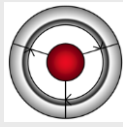




LGD and Adjusted Predicted LGD - Means by Quantile

Mean RR=0.8 and Mean LTV=0.8





Summary

- Models for Recovery-LGD ideally take LTV into account
 - Predictions based on a mean Recovery Rate that is calculated using sales proceeds without taking note of each case's LTV are biased
 - The degree of optimism will depend on the amount of overlap between LTV and RR distributions
 - Bias is stronger for smaller LGD values
- One should maintain models that predict if a case's LTV will exceed the RR and if so, what RR can be expected
 - Logistic regression of $(LTV > RR)$ against LTV
 - Linear regression of $(RR | LTV > RR)$ against LTV
- The improvements from removing bias should be weighed against the increase in model risk