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Monitoring Credit Portfolios using Survival Analysis

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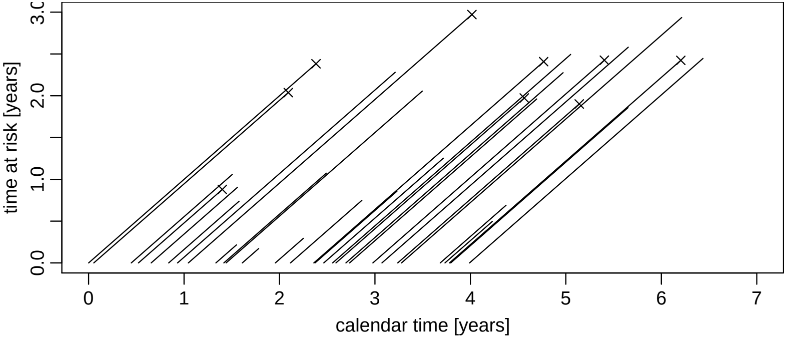
Setup

- Customers take out a loan at known times
- Repay for some time until
  - they finished repaying or
  - a default occurs.
- Lexis Diagramm:



Detecting Changes in the Defaults in a Credit Portfolio

- Main steps:
  - Construct model for in-control behaviour
  - Set up monitoring scheme
  - If alarm: investigate
- Use Survival Analysis (How to? Why?)
- Not restricted to defaults in credit portfolios: applies to other type of events (e.g. churn, purchase, fraud)

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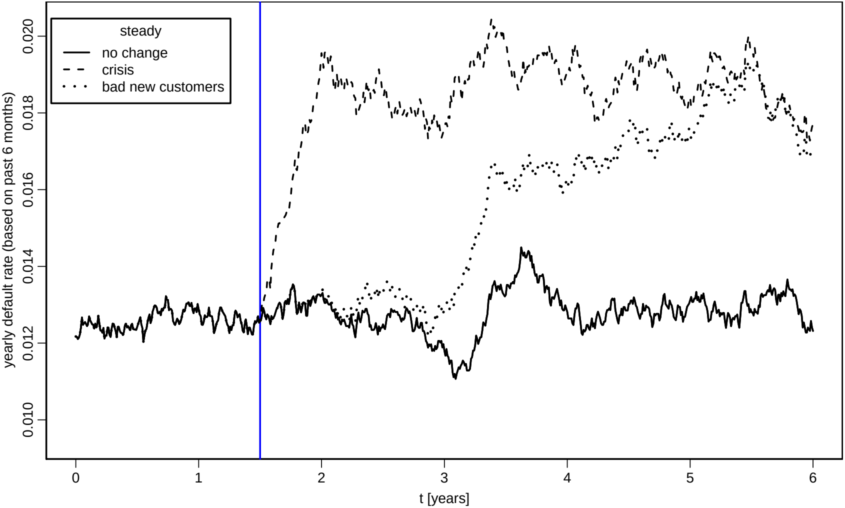
How do changes affect default rates? Simulation Example

- 5 year loans; early repayment possible
- Default distribution:
  Low constant hazard rate during first year,
  higher constant hazard rate after first year
- Arrival scenarios:
  - Steady (Poisson Process)
  - Doubling of new customers (at time 1)
  - Halving of new customers (at time 1)
- Scenarios for change in default rates:
  - No change
  - Crisis (at $t = 1.5$) - all customers affected
  - Bad new customers (at $t = 1.5$) - only new customers affected

Yearly Default Rates (6 month gliding window)



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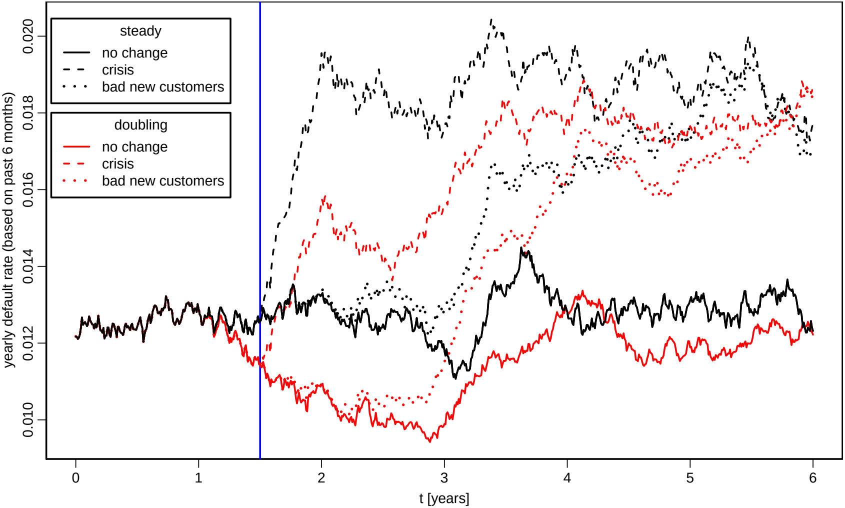

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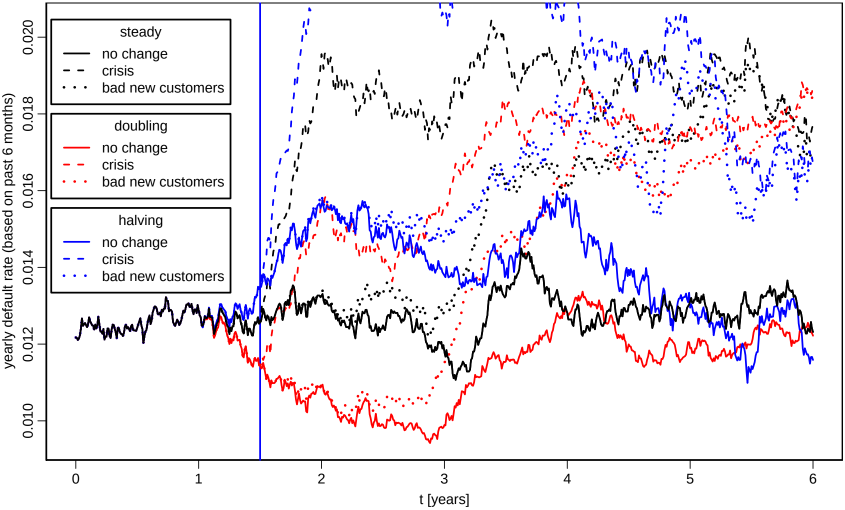
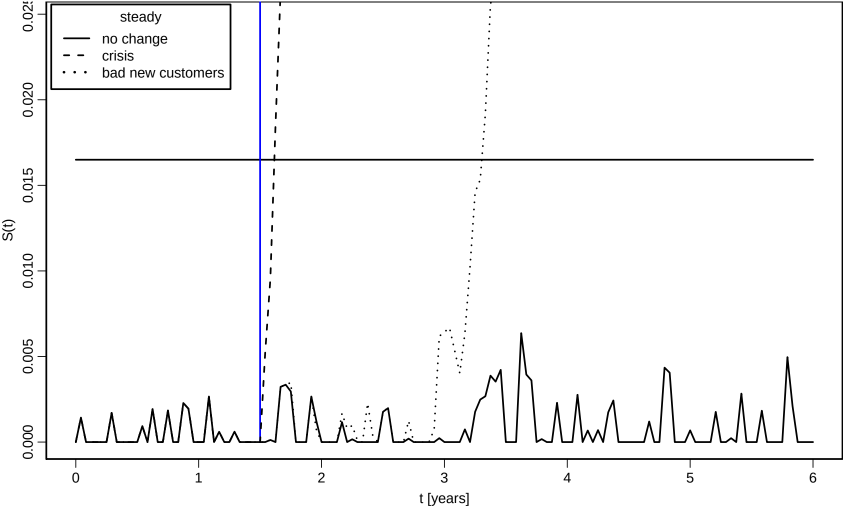


CUSUM charts (Page, 1954)

- Default rates $X_1, X_2, \ldots$
- $R_t = \sum_{i \leq t}(X_i - k)$
- $S_t = R_t - \min_{s \leq t} R_S$
- Signal at $\tau = \min\{t : S_t \geq c\}$

CUSUM charts (based on half-monthly default rates)



threshold such that P(false alarm in 10 years) ≤ 0.01

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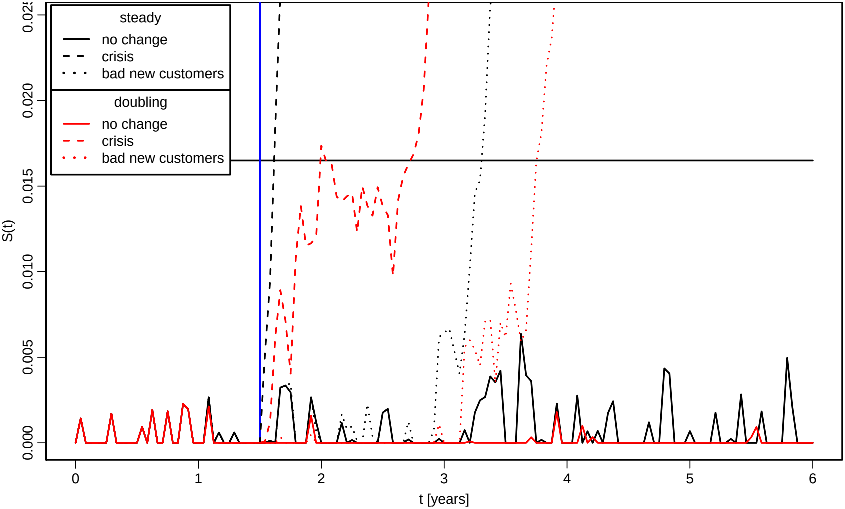

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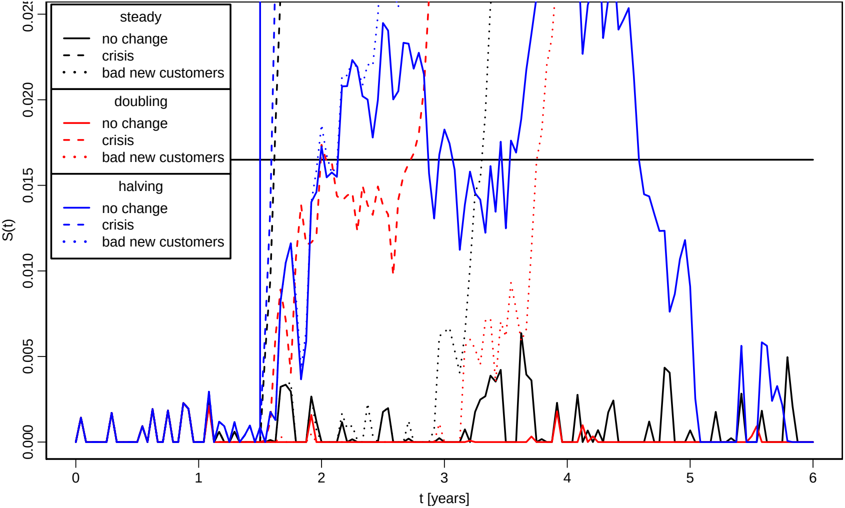

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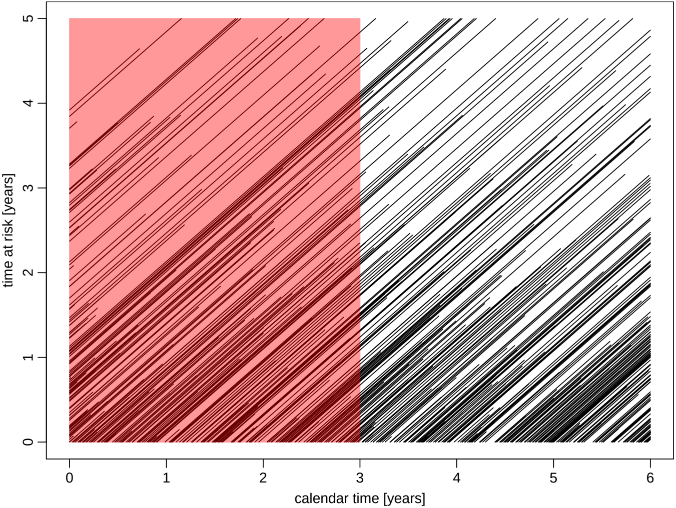
Monitoring Default Rates - Lexis Diagramm



Fixed Follow-Up

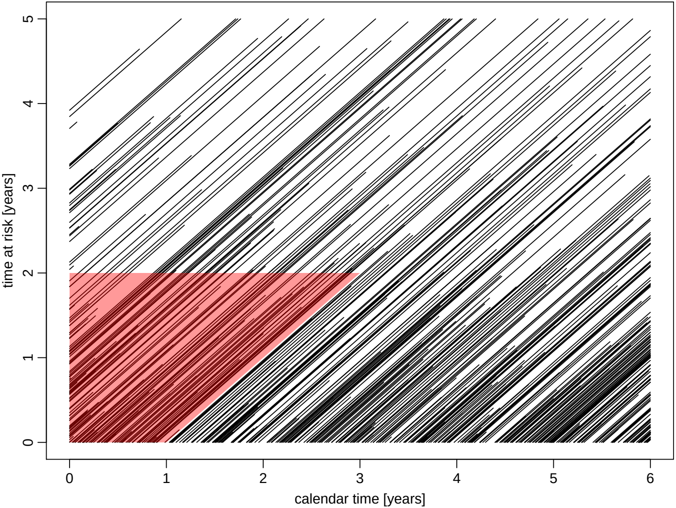
- Model for probability of default up to a given time t_0

$$P(T_i \leq t_0 | Z_i)$$

  (many choices, e.g. logistic regression)
- CUSUM chart based on likeklihood ratio between an alternative and the in-control model

Fixed Follow-Up- Lexis Diagramm

$t_0 = 2$ years, individuals considered at time $B_i + t_0$



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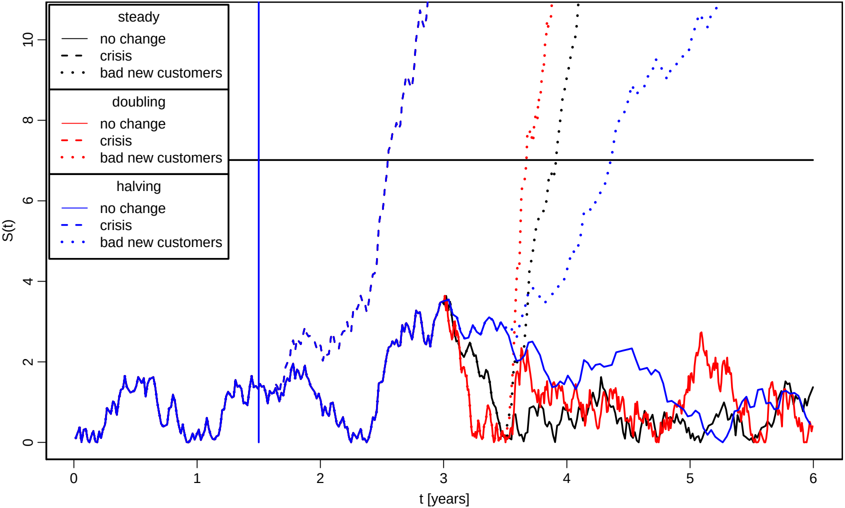
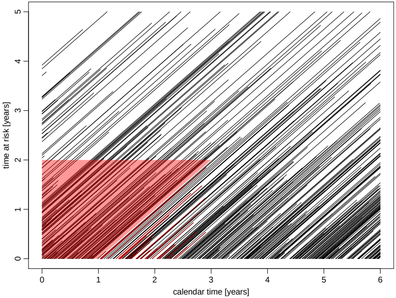


Fixed Follow-Up - Use Information Immediately?

- Not a good idea! $\rightarrow$ population effects!
- Example:
  - follow-up time $t_0 = 2$
  - arrival of new customers changes at $t = 1$
  - use default information about individuals at time $B_i + \min(X_i, t_0)$



- If no early repayment, i.e. no censoring:
  - # of defaults changes immediately
  - # of non-defaults changes only at $t = 3$
- With early repayment:
  - more complicated effects

Survival Analysis in Credit Scoring

- A lot of models to choose from, e.g. Cox's proportional hazard model.
- Use in credit risk, see e.g. Banasik et al. (1999), Stepanova & Thomas (2002, 2001)
- Early repayment (censoring) is dealt with automatically

Detecting a Proportional Change in the Hazard

$B_1 \leq B_2 \leq \dots$ calendar times at which individuals arrive
T_i survival time
C_i censoring time
$h_i(s)$ in control hazard rate of T_i
$\rho h_i(s)$ hazard rate after change point
$X_i(t) = \min(T_i, C_i, (t - B_i)^+)$ time at risk up to t
$\delta_i(t) = \mathrm{I}\{T_i \leq X_i(t)\}$ indicator for observed default up to t

$N(t) = \sum_i \delta_i(t)$ number of defaults up to time t
$\Lambda(t) = \sum_i \int_0^{X_i(s)} h_i(s)ds$ intensity in control up to time t
$\rho\Lambda(t)$ intensity out of control up to time t

Log-likelihood ratio :

$$R(t) = \log(\rho)N(t) - (\rho - 1)\Lambda(t)$$

$$S(t) = R(t) - \min_{0 \leq u \leq t} R(u); \qquad \tau = \min\{t : S(t) > c\}$$

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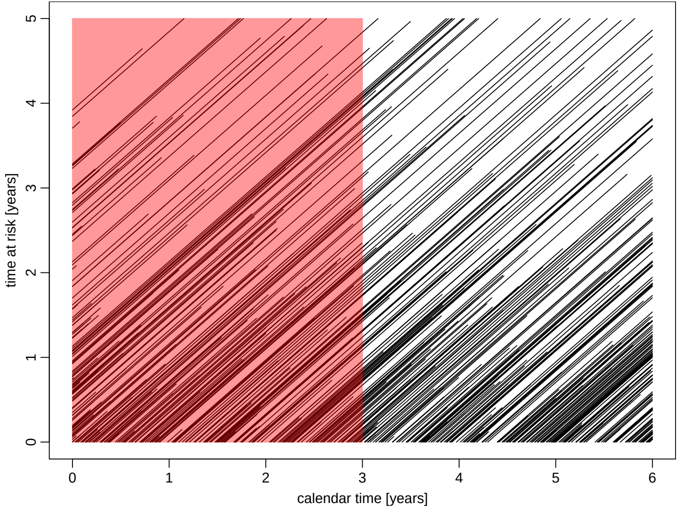

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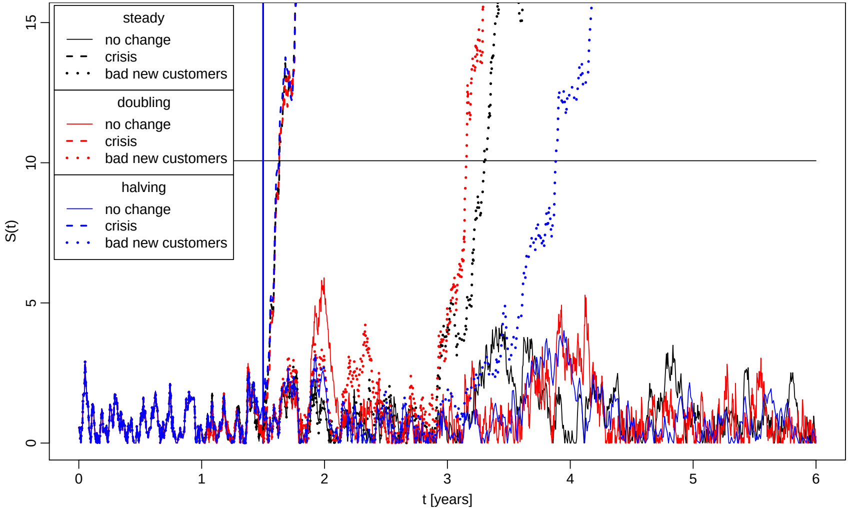

Hitting Times in the Example [months after change]

	steady		doubling		halving	
	crisis	bnc	crisis	bnc	crisis	bnc
Discrete CUSUM	1.5	22	6	27.5	1.5	6
Fixed Follow-Up	12.65	29.06	12.65	26.14	12.65	34.38
Surv. Anal. CUSUM	1.63	21.61	1.6	19.89	1.56	28.64

- Discrete CUSUM= direct monitoring of default rates

Comments

- General methodology for monitoring with Survival Analysis: Gandy et al. (2009)
- Describes e.g. how to set thresholds:
  Explicit computations possible for
  - $\mathrm{E}(N(\tau))$
  - $\mathrm{P}(N(\tau) \leq k)$
- Also possible: Monitoring against a decrease in events

Why use Survival Analysis for Monitoring?

- Quick detection (all available information is used)
- No problem with population effects

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