

Multiperiod Corporate Default Prediction – A Forward Intensity Approach

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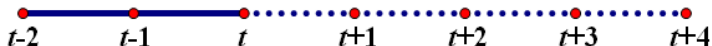
(August 2011)

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- 2 A forward intensity approach to multiperiod default prediction
 - Model setup
 - Estimating the forward intensity model
- 3 Data and covariates
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 - Comparison with Duffie, et al (2007)
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Research Question

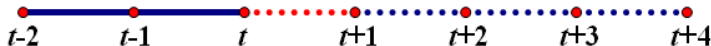
- Default/bankruptcy prediction over different future periods



- Term structures of default probabilities
 - Short-term vs. long-term
- A forward intensity approach
 - Reduced form model

Literature Review

- Discriminant analysis
 - Beaver (1966, 1968), Altman (1968), etc.
 - Model output: credit scores
- Binary response models: logit/probit regressions
 - Ohlson (1980), Zmijewski (1984), etc.
 - Model output: default probability in the next one period



- Campbell, *et al* (2008): logit models for different periods ahead

Literature Review (Cont'd)

- Recent development: duration analysis
 - Shumway (2001), Chava and Jarrow (2004), etc.
- Duffie, *et al* (2007): intensity model with stochastic covariates
 - Two Poisson processes
 - 1 Default/bankruptcy
 - 2 Other exit: merger and acquisition, etc.
 - Instantaneous intensity
 - Instantaneous rate of occurrence
 - Functions of the covariates
 - Time-series dynamics of the covariates

Model Setup

- Spot combined exit intensity: "average" rate of combined exit occurrence

$$\psi_{it}(\tau) \equiv -\frac{\ln(1 - F_{it}(\tau))}{\tau} = -\frac{\ln E_t \left[\exp \left(-\int_t^{t+\tau} (\lambda_{is} + \phi_{is}) ds \right) \right]}{\tau}$$

$F_{it}(\tau)$: the time- t conditional distribution function of the combined exit time evaluated at $t + \tau$.

λ_{is} : instantaneous intensity for default.

ϕ_{is} : instantaneous intensity for other exit.

Model Setup (Cont'd)

- Forward exit intensity: forward rate of combined exit occurrence

$$g_{it}(\tau) \equiv \frac{F'_{it}(\tau)}{1 - F_{it}(\tau)} = \psi_{it}(\tau) + \psi'_{it}(\tau)\tau$$

- Forward default intensity: forward rate of default occurrence

$$\begin{aligned} f_{it}(\tau) &\equiv e^{\psi_{it}(\tau)\tau} \lim_{\Delta t \rightarrow 0} \frac{P_t(t + \tau < \tau_{Di} = \tau_{Ci} \leq t + \tau + \Delta t)}{\Delta t} \\ &= e^{\psi_{it}(\tau)\tau} \lim_{\Delta t \rightarrow 0} \frac{E_t \left[\int_{t+\tau}^{t+\tau+\Delta t} \exp \left(- \int_t^s (\lambda_{iu} + \phi_{iu}) du \right) \lambda_{is} ds \right]}{\Delta t} \end{aligned}$$

τ_{Di} : default time of the i -th firm.

τ_{Ci} : combined exit time of the i -th firm.

- The default probability over $[t, t + \tau]$ becomes $\int_0^\tau e^{-\psi_{it}(s)s} f_{it}(s) ds$.

Model Setup (Cont'd)

- Model $f_{it}(\tau)$ and $g_{it}(\tau)$ directly as functions of state variables available at time t and the horizon of interest, τ .
- $g_{it}(\tau) \geq f_{it}(\tau)$
- $X_{it} = (x_{it,1}, x_{it,2}, \dots, x_{it,k})$: the set of the state variables

$$f_{it}(\tau) = \exp\{\lambda(\tau) \exp[-\delta(\tau)(t - t_B)] \mathbf{1}_{t > t_B} + \alpha_0(\tau) + \alpha_1(\tau)x_{it,1} + \alpha_2(\tau)x_{it,2} + \dots + \alpha_k(\tau)x_{it,k}\}$$

$$g_{it}(\tau) = f_{it}(\tau) + \exp(\beta_0(\tau) + \beta_1(\tau)x_{it,1} + \beta_2(\tau)x_{it,2} + \dots + \beta_k(\tau)x_{it,k})$$

where t_B is August 2008 (Note that the US government bailed out AIG in September 2008)

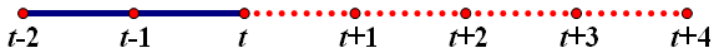
- Discretize the model for empirical implementation

Model Setup (Cont'd)

- Forward default probability



- Cumulative default probability



Estimating the Forward Intensity Model

- Maximize an overlapped likelihood function.
- The likelihood function is decomposable so that estimation can be performed one forward period at a time.
- Because the numerical problem is non-sequential, it can be easily parallelized in computing.

Data

- Sample period: 1991-2010.
- Database:
 - Compustat
 - CRSP
 - Bloomberg
- 12,225 U.S. public companies (both industrial and financial), 1,066,337 firm-month observations.

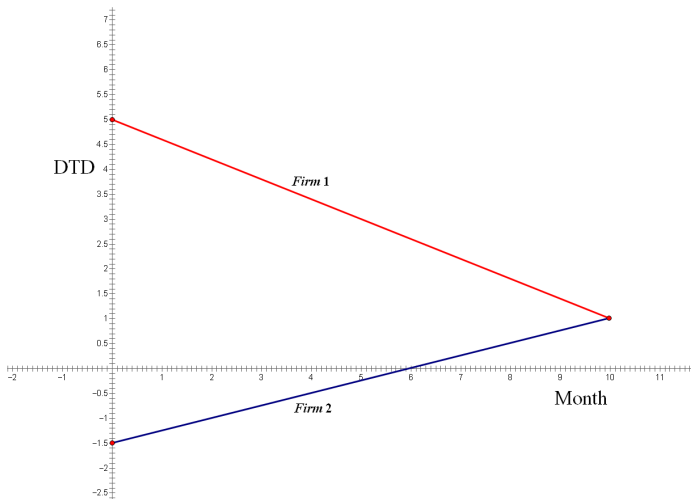
Year	Active Firms	Defaults	(%)	Other Exit	(%)
1991	4018	28	0.70%	258	6.42%
1992	4016	23	0.57%	333	8.29%
1993	4194	16	0.38%	213	5.08%
1994	4428	13	0.29%	281	6.35%
1995	5064	18	0.36%	396	7.82%
1996	5453	16	0.29%	472	8.66%
1997	5641	46	0.82%	553	9.80%
1998	5705	66	1.16%	749	13.13%
1999	5423	73	1.35%	745	13.74%
2000	5086	102	2.01%	616	12.11%
2001	4912	156	3.18%	583	11.87%
2002	4675	80	1.71%	403	8.62%
2003	4336	62	1.43%	370	8.53%
2004	4073	24	0.59%	306	7.51%
2005	3920	23	0.59%	292	7.45%
2006	3852	19	0.49%	283	7.35%
2007	3764	21	0.56%	353	9.38%
2008	3672	57	1.55%	290	7.90%
2009	3581	70	1.95%	246	6.87%
2010	3385	32	0.95%	228	6.74%

Covariates

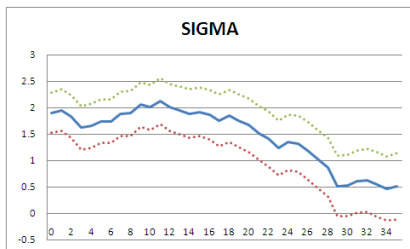
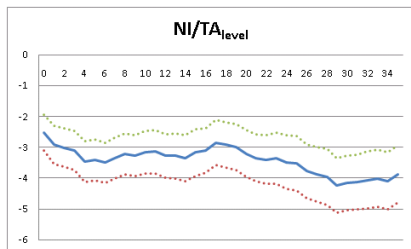
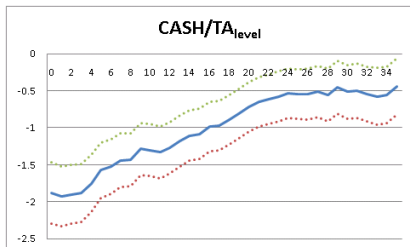
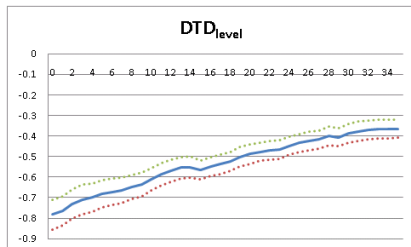
- An exponential decaying term to capture the US intervention effect
- 3-month treasury rate
- Trailing 1-year S&P500 return
- Distance to default
- Cash and short-term investments/Total assets
- Net income/Total assets
- Relative size
- Market to book ratio
- Idiosyncratic volatility

Covariates (Cont'd)

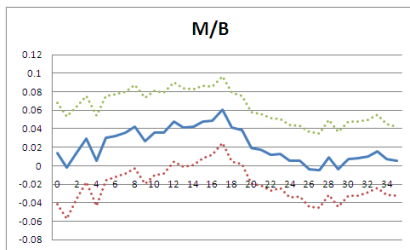
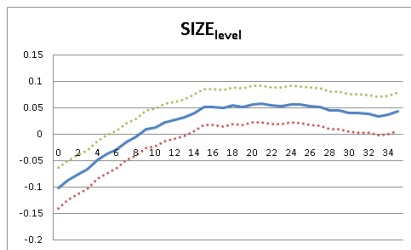
- Level and trend



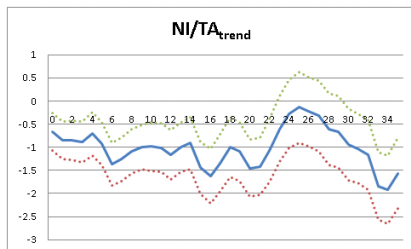
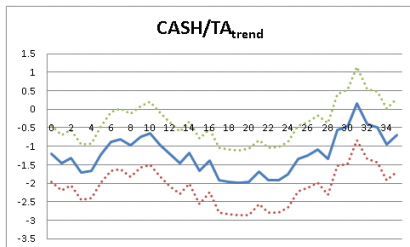
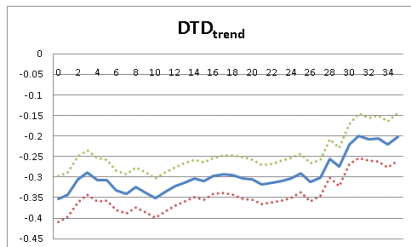
Parameter Estimates



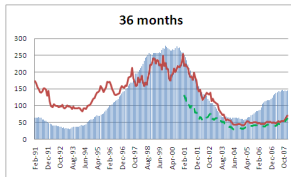
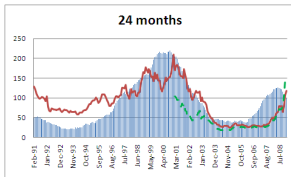
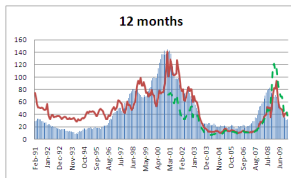
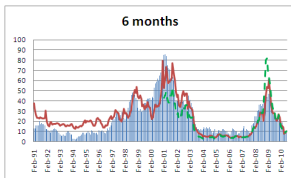
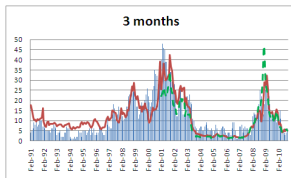
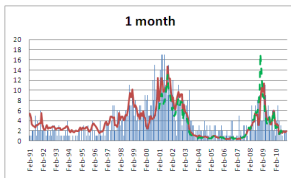
Parameter Estimates (Cont'd)



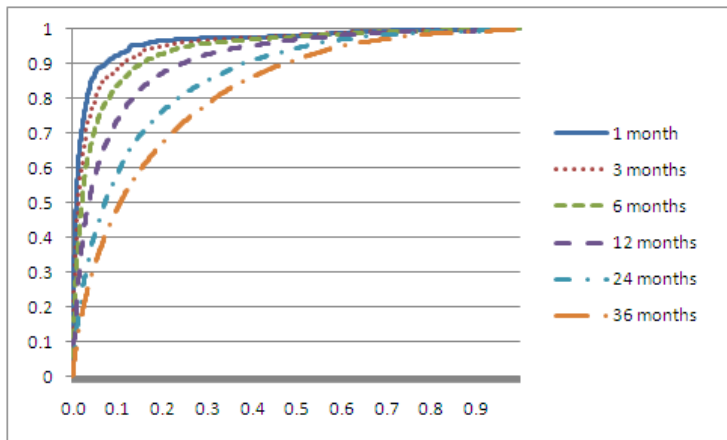
Parameter Estimates (Cont'd)



Aggregate Number of Defaults

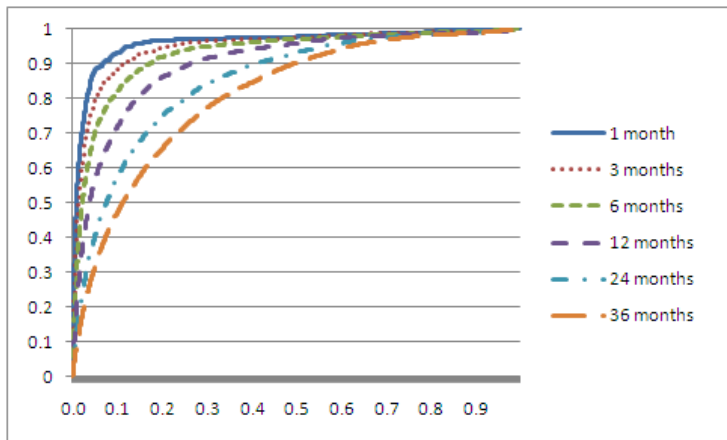


In-Sample Accuracy



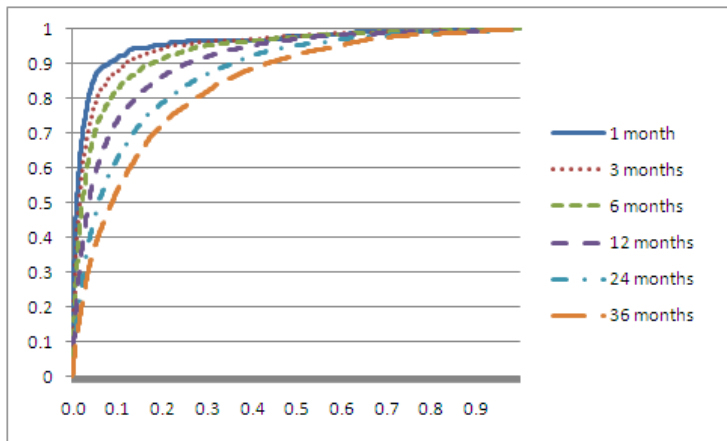
1 month	3 months	6 months	12 months	24 months	36 months
92.81%	90.85%	88.06%	82.70%	73.44%	65.69%

Out-of-Sample (Cross-Section) Accuracy



1 month	3 months	6 months	12 months	24 months	36 months
92.42%	89.89%	86.42%	80.69%	71.48%	64.00%

Out-of-Sample (Over Time) Accuracy



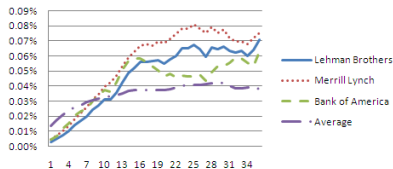
1 month	3 months	6 months	12 months	24 months	36 months
91.74%	90.06%	87.24%	82.39%	75.90%	69.80%

Accuracy Ratios with/without Smoothing

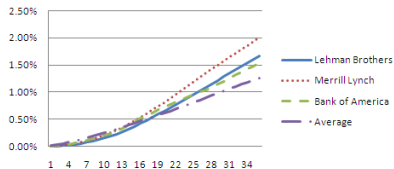
Panel A: In-sample result						
	1 month	3 months	6 months	12 months	24 months	36 months
Full sample	92.81%	90.85%	88.06%	82.70%	73.44%	65.69%
Full sample (smoothed)	92.88%	90.91%	88.11%	82.67%	73.42%	65.67%
Non-financial	92.52%	90.33%	87.34%	81.78%	73.00%	65.92%
Financial	95.17%	94.49%	92.92%	89.07%	76.07%	58.32%
Panel B: Out-of-sample (cross-section) result						
	1 month	3 months	6 months	12 months	24 months	36 months
Full sample	92.42%	89.89%	86.42%	80.69%	71.48%	64.00%
Full sample (smoothed)	92.12%	89.75%	86.46%	80.73%	71.45%	64.00%
Non-financial	92.05%	89.35%	85.57%	79.66%	71.23%	64.49%
Financial	98.02%	96.64%	95.63%	90.51%	75.85%	55.62%
Panel C: Out-of-sample (over time) result						
	1 month	3 months	6 months	12 months	24 months	36 months
Full sample	91.74%	90.06%	87.24%	82.39%	75.90%	69.80%
Full sample (smoothed)	91.71%	90.10%	87.18%	82.26%	76.00%	69.78%
Non-financial	91.70%	89.94%	87.02%	82.03%	75.99%	70.18%
Financial	94.61%	93.81%	91.90%	88.79%	81.57%	71.62%

Forward and Cumulative Term Structures of PDs

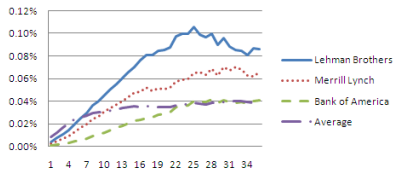
Forward default probability
(36 months before bankruptcy)



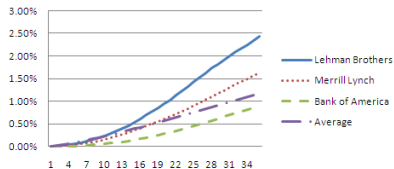
Cumulative default probability
(36 months before bankruptcy)



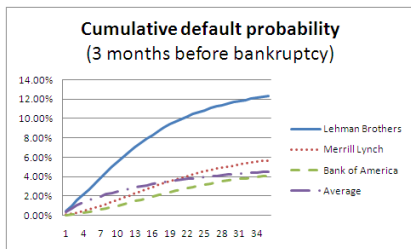
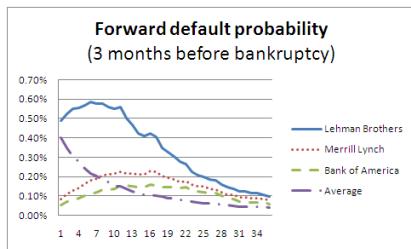
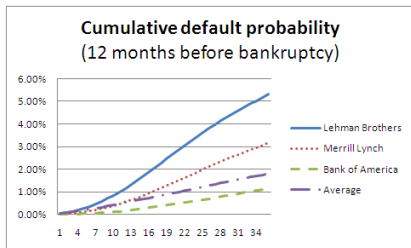
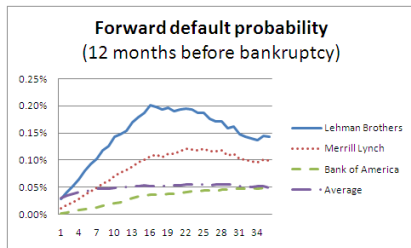
Forward default probability
(24 months before bankruptcy)



Cumulative default probability
(24 months before bankruptcy)



Forward and Cumulative Term Structures of PDs (Cont'd)



Comparison with Duffie, *et al* (2007)

Use 3000 non-financial firms (randomly sampled from 12,225 firms) and apply 4 covariates (trailing one-year S&P 500 index return, 3-month treasury rate, firms' distance-to-default and firms' trailing one-year stock return)

Panel A: In-sample result (1991-2010)						
	1 month	3 months	6 months	12 months	24 months	36 months
Duffie, <i>et al</i> (2007)	90.56%	89.29%	87.68%	85.17%	81.62%	79.60%
Forward Intensity	90.56%	88.65%	85.71%	80.65%	71.85%	65.91%
Panel B: In-sample result (2001-2010)						
	1 month	3 months	6 months	12 months	24 months	36 months
Duffie, <i>et al</i> (2007)	90.96%	90.28%	88.62%	87.67%	83.46%	79.42%
Forward Intensity	90.96%	89.96%	87.29%	85.01%	78.73%	73.84%
Panel C: Out-of-sample (over time) result (2001-2010)						
	1 month	3 months	6 months	12 months	24 months	36 months
Duffie, <i>et al</i> (2007)	91.21%	90.12%	87.08%	81.80%	69.05%	62.19%
Forward Intensity	91.21%	90.36%	87.81%	85.51%	78.63%	74.25%

Conclusion

- A forward intensity approach for the prediction of corporate defaults over different future periods is proposed.
- Maximum likelihood analysis is conducted on a large sample of U.S. industrial and financial firms spanning the period 1991-2010.
- Several frequently used covariates are shown to be useful for prediction at both short and long horizons.
- The results confirm the bailout effect and capture the Lehman Brothers episode remarkably well.
- The forward intensity model is amenable to aggregation, which allows analysts to assess default behavior at the portfolio and/or economy level.