Scoring Decisions under Basel II Constraints

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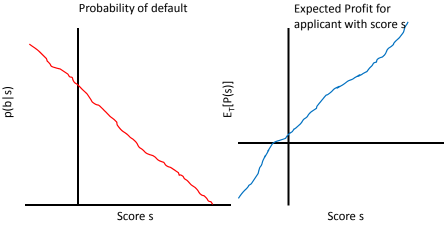
Introduction

- Suppose a portfolio manager has access to a pool of loan applicants and a scorecard to score these applicants.
- In making a score decision, portfolio managers use scores to
  - evaluate potential applicants on one or more of the bank's objectives, and
  - make a decision on loan applications, i.e., accept or decline.
- In this study, we incorporate regulatory capital requirements into the decision making process.
- We assume the portfolio manager has access to a pool of capital for regulatory purposes.
- When making these accept/reject decisions, portfolio managers must also ensure the consumer loan portfolio is adequately capitalized,
  - i.e., decide on how much of the pool of capital to retain for regulatory purposes.

Bank Objectives, scores and Controls

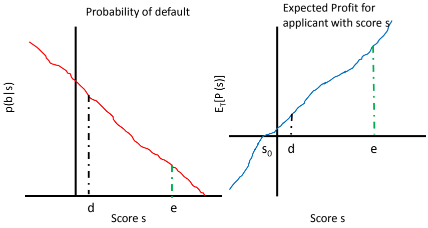
- The portfolio managers objectives are to:
  - maximize profit,
  - maximize market share (volume), and
  - minimize losses.
- In our study, we will restrict attention to the business objectives of profit, denoted by P, and volume, denoted by V.
- The portfolio manager has access to credit scores on each applicant with higher score applicants having lower probability of default.
- The portfolio manager is required to
  - make an accept/reject decision on each applicant, and
  - ensure the portfolio is adequately capitalized.
- Our goal is to understand the structure that is shared by every efficient acquisition policy under regulatory requirements.

Consumer Loan Economics



- The expected profit for an applicant with score s is $E_T[P(s)] = (r_L - c_D)p(g|s) - (l_D + c_D)p(b|s)$,
  - T is the outcome of each account (b indicating an account that defaults and g indicating a good account),
  - r_L is the rate of return on a good account,
  - l_D is the loss given default for a bad account, and
  - c_D is cost of debt with $c_D < r_L$.

Single Cutoff Score and Profit Maximization Policy

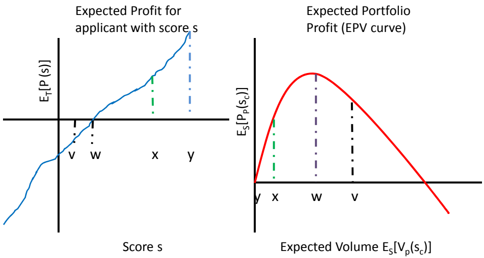


- Since lower score is associated with higher probability of default, expected profit, $E_T[P(s)]$ monotonically increases with score.
- Suppose a portfolio manager's objective is to maximize expected profit.
- The portfolio manager will accept all those scores with non-negative expected profit,
  - accept all scores above the cutoff score s_0, i.e., $s \geq s_0$, and
  - reject scores below cutoff score s_0, i.e., $s < s_0$,
  - such that $E_T[P(s_0)] = 0$.

Expected Portfolio Profit-Volume (EPV) Curve

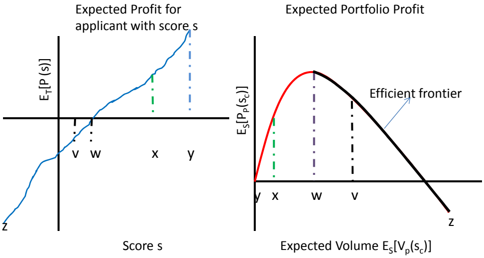
- Suppose instead the portfolio manager operates under the objectives of
  - maximizing expected portfolio profit, and
  - maximizing expected portfolio volume.
- Due to the well-ordered nature of the score-set, the portfolio manager will operate under a single cutoff-score policy.
- Suppose $E_S[P_p(s_c)] \equiv E_S[E_T[P_p(s_c)]]$ is the expected portfolio profit with cutoff score s_c
  - Similarly, $E_S[V_p(s_c)] \equiv E_S[E_T[V_p(s_c)]]$ is the expected portfolio volume with cutoff score s_c
- We term the collection of points $(E_S[P_p(s_c)], E_S[V_p(s_c)])$ for cutoff-score $s_c \in (-\infty, \infty)$, an Expected-Profit-Volume (EPV) curve.

Expected Portfolio Profit-Volume (EPV) Curve



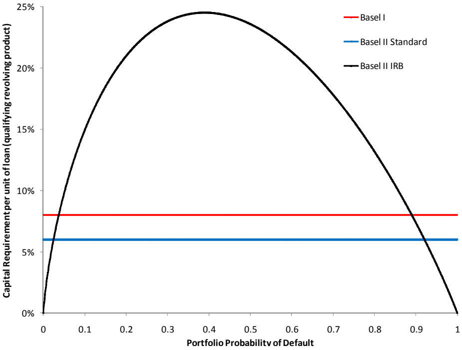
- Initially, when s_c is high, no applicant is accepted, i.e., $E_S[P_p(s_c)] = 0$ and $E_S[V_p(s_c)] = 0$ (y).
- As cutoff score s_c is lowered, the expected profit increases (x) until maximum portfolio profit is reached (w).
- With further decrease in s_c, expected profit decreases as expected volume increases (v).

Efficient Frontier



- We call the maximal set of operating points that are not dominated by other operating points, *an efficient frontier* [1,2].
- For a given level of expected profit, the maximum expected volume is achieved on the EPV curve $\bar{wz}$.
- A decision may be made on $\bar{wz}$ based on the bank's trade-offs.
- However, the profit function used above does not include cost of regulatory capital.

Historical Basel Requirements

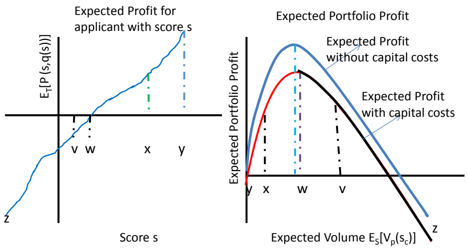


- The first Basel Accord specified a constant regulatory capital requirement of a constant 8% per dollar of credit.
- Basel II gives financial institutions an option to choose between a constant rate of 6% and a risk based formula, i.e., internal rating based method or Basel II IRB.

The Profit Function

- We incorporate the cost of regulatory capital to the profit function, i.e., $E_T[P(s, q(s))] = E_T[P(s)] - r_Q q(s) = (r_L - c_D)p(g|s) - (f_D + c_D)p(b|s) - r_Q q(s)$.
  - $q(s)$ is the regulatory capital requirement for an account with score s.
  - r_Q is the rate of return expected by the shareholders.
- Let us first consider the case of a portfolio manager without capital constraints.

Efficient Frontier

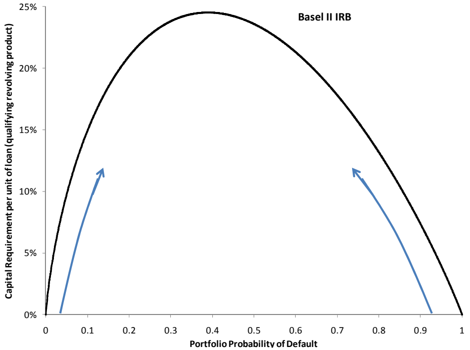


- Expected profit, $E_T[P(s, q(s))]$ increases with respect to score s.
- As in the previous section, for the case of unlimited capital access, all efficient policies are constructed through single cutoff score strategy.
- Note, economic capital requirement increases along expected volume axis.

Capital Constraint Case

- Suppose the portfolio manager is constrained by the amount of capital provided by the shareholders, i.e., Q_T.
- We restrict our study to the more interesting case of a portfolio manager operating under Basel II IRB.
- We provide a motivating example for the study of the capital constraint case.

Motivating Example

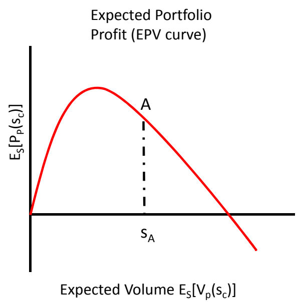


- Suppose the portfolio manager wished to maximize volume.
- The volume maximizing strategy would be to accept all scores with the lowest capital requirement, until the capital constraint is binding.
  - Accept those scores at both end of the risk spectrum.
- Clearly, the optimal strategy is not a single cutoff score strategy.

Simulation

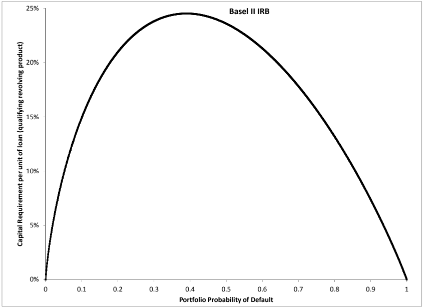
- We could use simulation as a solution method to determine the efficient frontier.
- For each instance of the simulation:
  - simulate for an accept set ω.
  - if the capital requirement for a portfolio with accept set ω is less than Q_T,
    - determine portfolio profit $E_S[P(\omega, q(s))]$ and portfolio volume $E_S[V(\omega)]$, and
    - plot the point on the EPV space.
- The accept set ω, would be a union of multiple disjoint score intervals.
- This type of simulation exercise would be inefficient due to the choice of ω.
- Our goal is to construct the efficient frontier in a more systematic way.

EPV Curve for the Capital Constraint Case

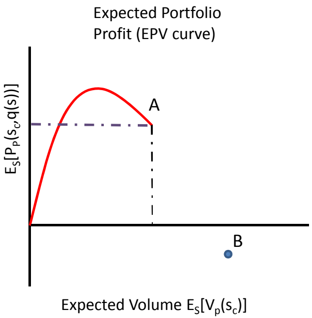


- Suppose $Q(s_c)$ is the capital requirement for a portfolio with cutoff score s_c.
- Suppose score s_A is such that $Q(s_A) = Q_T$.
- Since $Q(s_c) \leq Q_T$ for $s_c \geq s_A$, all operating points left of point A on the unconstrained EPV curve is part of the EPV curve for the capital constraint case.

Maximum Volume Efficient Point

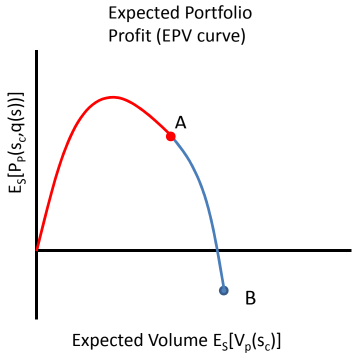


- Maximum volume point on the EPV curve is constructed as follows:
  - accept scores with the least capital requirements, i.e., $\omega_B = (-\infty, s_1] \cup [s_2, \infty)$ and $q(s_1) = q(s_2)$, and
  - accept as many as possible until the capital constraint is binding, i.e., $Q(s_2) + [Q(-\infty) - Q(s_1)] = Q_T$.



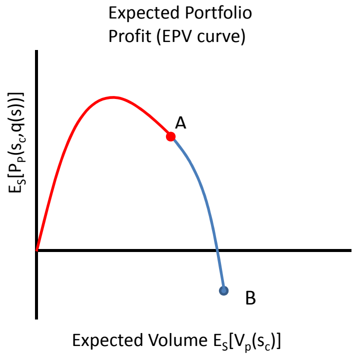
- Suppose point B is the volume maximizing point on the EPV curve.
- The operating points on the EPV curve between points A and B are constructed by excluding those accept sets with lower expected profit than other accept sets with equal expected volume.

EPV Curve for the Capital Constraint Case



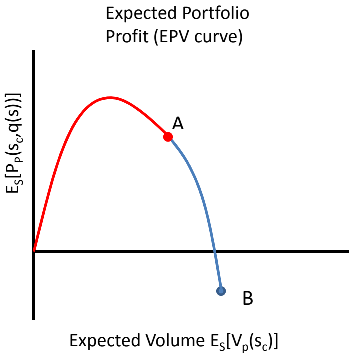
- The accept set for all points on the EPV curve between A and B are in the form:
  - $\omega = (-\infty, s_x] \cup [s_y, \infty)$, such that
  - $q(s_y) > q(s_x)$, and
  - $Q(s_y) + [Q(-\infty) - Q(s_x)] = Q_T$, i.e., capital constraint is binding.
  - We call a score set of this form, a double cutoff-score set.

EPV Curve for the Capital Constraint Case



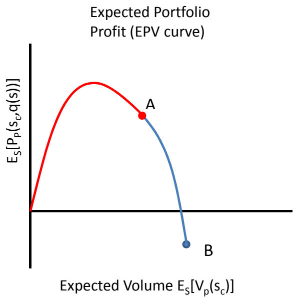
- Operating points from point A to point B are plotted as follows:
  - set an initial value of $s_y = s_A$, where $Q(s_A) = Q_T$,
  - increase s_x and s_y in a manner such that $Q(s_y) + [Q(-\infty) - Q(s_x)] = Q_T$,
  - stop when $q(s_y) = q(s_x)$.

EPV Curve



- The resulting EPV curve for the constrained problem is concave.
- If $E_S[P_n(-\infty, q(s))] < 0$, then the double cutoff-score portion of the EPV curve is decreasing.
- Since we have constructed the EPV curve, we may determine the efficient frontier.

EPV Curve



- As we move along the EPV curve from point A to point B, we replace scores in the accept sets by the riskiest scores from the non-accept set.
- This has the effect of increasing the risk profile of the portfolio.
- However, the capital requirement for all operating points between point A and B remains constant at Q_T.
- This may create a false sense of security with respect to portfolio default risk since the capital requirement is constant.

Multiple Economics Scenario

- Thus far, we considered the case of a portfolio manager access to one scorecard.
- The scorecard forecasts default probabilities under the assumption of a single economic scenario.
- Suppose instead, the portfolio manager has access to multiple scorecards.
- Each scorecard built to predict default outcomes in different economic scenarios (see [3]).
- Suppose the portfolio manager has access to forecasted probabilities of occurrence of each scenario.

Scoring Decision in Multiple Economic Scenarios

- The scoring decision is dependent on the prevailing economic conditions during the loan period.
- However, the policy must be specified and implemented before the loan period and hence before the economic environment is known with certainty.
- Suppose the portfolio manager is required to ensure the portfolio is adequately capitalized for either economic scenario.
- Our framework for constructing the EPV curve for the capital constraint case lends itself to constructing the efficient frontier in the multiple economic scenario case.
- This result increases the efficiency of the simulation for the multiple economic scenario case.

Final Remarks

- In this work, we considered the objectives of maximizing expected profit and volume.
- We incorporated the cost of regulatory capital and considered the cases of both unlimited and limited access to capital.
- We showed that for the case of unlimited access to capital, all efficient operating points are constructed through a single cutoff-score strategy.
- For the case of limited access to capital, efficient operating points are constructed through the combination of
  - a single cutoff-score strategy, and
  - a double cutoff-score strategy.
- This result allows us to increase the efficiency of the simulation when constructing the efficient frontier for the multiple economic scenario case.

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