

Stress Testing Credit Risk Parameters

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Agenda

- 1 Stress Testing and Credit Risk
- 2 Framework
- 3 Empirical Study
- 4 Conclusion

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Is There a Need for Stress Testing?

- Banks use credit portfolio models for economic capital
- Basel II: Internal Ratings Based Approach (IRBA) for regulatory capital
- Formula is based on 'stressed' assumptions:
 - 99.9 % confidence level for systematic factor (similar to Value-at-Risk)
 - Conservative (high) values for asset correlations (compare eg. Dietsch/Petey, 2004, Rösch/Scheule, 2004, 2005)
 - 'Downturn' estimates for LGD and EAD

Is There a Need for Stress Testing?

- IRBA accompanied by stress tests (Basel Committee, p.5):

'A bank must have in place sound stress-testing processes for use in the assessment of capital adequacy. These stress measures must be compared against the measure of expected positive exposure and considered by the bank as part of its internal capital adequacy assessment process. Stress-testing must also involve identifying possible events or future changes in economic conditions that could have unfavorable effects on a firm's credit exposures and assessment of the firm's ability to withstand such changes. Examples of scenarios that could be used are:

- 1 Economic or industry downturns,
- 2 Market-place events, or
- 3 Decreased liquidity conditions.

In addition to the more general tests described above, the bank must perform a credit risk stress-test to assess the effect of certain specific conditions on its IRB regulatory capital requirements[...]

Is There a Need for Stress Testing?

- Lopez (2005): categorizes stress tests into
 - Sensitivity analysis: Assessment of the impact of a large move in financial/macroeconomic variables
 - Scenario analysis: Assessment of the impact of unlikely events (eg 9/11).
- BIS (2005): Credit risk stress testing
 - Trading book: Similar to market risk stress testing
 - Banking book: Stress scenarios of PDs and LGDs

Is There a Need for Stress Testing?

- Berkowitz (2000): Basic risk model should already include all possible scenarios (eg. VaR)
 - ⇒ No need for stress testing ?
 - ⇒ If stress testing is done, rather include the scenarios in the risk model
 - ⇒ Basic problem of stress testing: No probabilities given for stress scenarios

This Paper

- We consider mechanics which are not captured by usual risk models
- Simultaneously stress the relevant parameters (PD and correlation)
- Explicitly assign probabilities to the stress scenario

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The credit risk model

- Obligor default

$$D_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults in } t \\ 0 & \text{otherwise} \end{cases}$$

$$(i \in N_t, t = 1, \dots, T)$$

- Threshold model with latent asset return

$$R_{it} < c \Leftrightarrow D_{it} = 1$$

The credit risk model

- Factor model for asset return

$$R_{it} = \omega F_t + \sqrt{1 - \omega^2} U_{it}$$

- Default barrier may depend on macroeconomic variables ('Point in Time')

$$c_t = \alpha + \beta' z_{t-1}$$

The credit risk model

- Conditional probability of default (CPD), given f_t and the economy

$$\pi(f_t, z_{t-1}) = P(D_{it} = 1 | f_t, z_{t-1}) = \Phi\left(\frac{\alpha + \beta' z_{t-1} - \omega f_t}{\sqrt{1 - \omega^2}}\right)$$

- Unconditional probability of default (PD) - depends on macroeconomy

$$\pi_t = P(D_{it} = 1 | z_{t-1}) = \Phi(\alpha + \beta' z_{t-1})$$

- 'Asset' correlation

$$\text{Corr}(R_{it}, R_{jt}) = \omega^2$$

Parameter Estimation

- Log-likelihood

$$l(\alpha, \beta, \omega) = \sum_{t=1}^T \ln \int_{-\infty}^{\infty} \left\{ \prod_{i \in N_t} \pi(f_t, \mathbf{z}_{t-1})^{d_{it}} (1 - \pi(f_t, \mathbf{z}_{t-1}))^{1-d_{it}} \right\} d\Phi(f_t)$$

- Solved numerically by adaptive Gauss-Hermite quadrature (Pinheiro/Bates, 1995, Rabe-Hesketh et al., 2002)
- Estimators asymptotically exist, are consistent and asymptotically normally distributed (Davidson/MacKinnon, 1993)
- Applications include: Gordy/Heitfield (2002), Hamerle et al. (2006), Rösch/Scheule (2004, 2005)

Stress Testing

- Credit portfolio loss of the risk model

$$L_{T+1} = \sum_{i=1}^{n_{T+1}} LGD_{iT+1} \cdot E_{iT+1} \cdot 1_{\{R_{iT+1} \leq c_t\}}$$

- Empirical credit portfolio loss

$$\hat{L}_{T+1} = \sum_{i=1}^{n_{T+1}} LGD_{iT+1} \cdot E_{iT+1} \cdot 1_{\{\hat{R}_{iT+1} \leq c_t\}}$$

- Forecast for PD

$$\hat{\pi}_{T+1} = \hat{P}(D_{iT+1} = 1 | z_T) = \Phi(\hat{\alpha} + \hat{\beta}' z_T)$$

Stress Testing

- Stressed PD

$$\pi_{stress,T+1} = \Phi(\alpha_{stress} + \beta'_{stress} z_T)$$

- Credit portfolio loss under stress

$$L_{stress,T+1} = \sum_{i=1}^{n_{T+1}} LGD_{iT+1} \cdot E_{iT+1} \cdot 1_{\{R_{stress,iT+1} \leq C_{stress,T+1}\}}$$

Stress Testing

- Simultaneously stress parameters using confidence intervals (eg, univariate via Bonferroni or multivariate)

$$P \left(\bigcap_j \left\{ \hat{\theta}_j - t \left(n-1; 1 - \frac{\gamma_j}{2} \right) \hat{s}(\hat{\theta}_j) \leq \theta_j \leq \hat{\theta}_j + \dots \right\} \right) \geq 1 - \gamma$$

$$P \left\{ (\hat{\theta} - \hat{\theta}_{stress})' \Sigma^{-1} (\hat{\theta} - \hat{\theta}_{stress}) \leq \frac{(n-1)J}{(n-J)n} F(J; n-J; 1-\gamma) \right\} \geq 1 - \gamma$$

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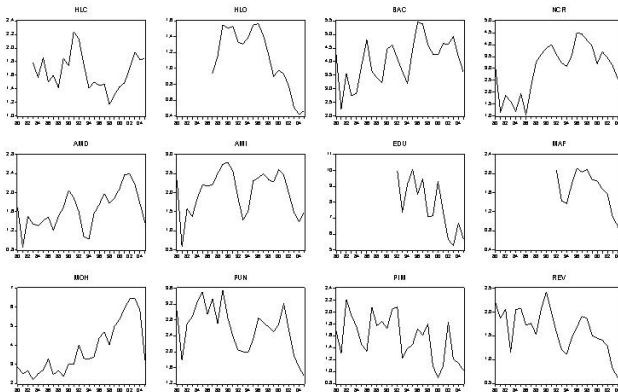
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Data

Loan category	Abbreviation	Exposure classes	Periods
Home equity loans (closed end)	HLC	Residential mortgage	1983-2005
Home equity loans (open end)	HLO		1987-2005
Bank card	BAC	Qualifying revolving	1980-2005
Non-card (revolving)	NCR		1980-2005
Automobile, direct	AMD	Other retail	1980-2005
Automobile, indirect	AMI		1980-2005
Education	EDU		1992-2005
Marine Financing	MAF		1992-2005
Mobile Home	MOH		1980-2005
Personal, unsecured	PUN		1980-2005
Property improvement	PIM		1980-2005
Recreational Vehicle	REV		1980-2005

Data

Abbildung: Delinquency rates of different loan categories



Parameter Estimates - Through the Cycle

Parameter	PD	$\hat{\alpha}$	$\hat{\alpha}$ SE	AC	$\hat{\omega}$	$\hat{\omega}$ SE
HLC	0.016	-2.133	0.013	0.004	0.062	0.010
HLO	0.011	-2.275	0.037	0.023	0.152	0.025
BAC	0.040	-1.746	0.019	0.009	0.096	0.014
NCR	0.030	-1.882	0.036	0.031	0.176	0.024
AMD	0.016	-2.133	0.020	0.010	0.100	0.014
AMI	0.020	-2.050	0.028	0.018	0.135	0.019
EDU	0.076	-1.433	0.030	0.011	0.107	0.021
MAF	0.016	-2.150	0.037	0.017	0.130	0.025
MOH	0.038	-1.776	0.031	0.023	0.151	0.021
PUN	0.026	-1.948	0.021	0.010	0.101	0.014
PIM	0.016	-2.154	0.020	0.009	0.097	0.014
REV	0.016	-2.151	0.029	0.019	0.139	0.019

Results - Through the Cycle

	HLC	HLO	BAC	NCR	AMD	AMI	EDU	MAF	MOH	PUN	PIM	REV
Expected loss												
no stress	0.017	0.011	0.040	0.030	0.016	0.020	0.076	0.016	0.038	0.026	0.016	0.016
stress = 0.10	0.018	0.014	0.044	0.035	0.018	0.023	0.086	0.019	0.043	0.028	0.017	0.018
stress = 0.05	0.018	0.015	0.045	0.036	0.019	0.024	0.088	0.020	0.044	0.029	0.018	0.019
stress = 0.01	0.018	0.016	0.046	0.038	0.019	0.025	0.092	0.022	0.046	0.030	0.018	0.020
stress = 0.001	0.019	0.017	0.048	0.041	0.020	0.026	0.098	0.024	0.049	0.031	0.019	0.021
Value-at-Risk												
no stress	0.026	0.034	0.073	0.087	0.033	0.050	0.134	0.039	0.093	0.050	0.031	0.041
stress = 0.10	0.032	0.055	0.091	0.128	0.044	0.070	0.182	0.064	0.129	0.065	0.041	0.059
stress = 0.05	0.033	0.059	0.095	0.135	0.046	0.073	0.192	0.069	0.135	0.068	0.043	0.062
stress = 0.01	0.035	0.070	0.102	0.152	0.050	0.082	0.215	0.082	0.150	0.073	0.047	0.070
stress = 0.001	0.038	0.087	0.112	0.177	0.056	0.094	0.251	0.106	0.171	0.082	0.052	0.081
BII Value-at-Risk												
no stress	0.155	0.121	0.125	0.099	0.114	0.124	0.201	0.112	0.153	0.135	0.112	0.112
stress = 0.10	0.162	0.139	0.133	0.113	0.123	0.136	0.220	0.129	0.169	0.144	0.120	0.124
stress = 0.05	0.164	0.142	0.135	0.115	0.124	0.138	0.224	0.132	0.172	0.146	0.122	0.126
stress = 0.01	0.166	0.149	0.138	0.120	0.127	0.143	0.232	0.140	0.177	0.149	0.125	0.131
stress = 0.001	0.170	0.159	0.142	0.127	0.132	0.149	0.244	0.151	0.185	0.154	0.129	0.137

Asset Correlations - Through the Cycle

	HLC	HLO	BAC	NCR	AMD	AMI	EDU	MAF	MOH	PUN	PIM	REV
no stress	0.004	0.023	0.009	0.031	0.010	0.018	0.012	0.017	0.023	0.010	0.010	0.019
stress = 0.10	0.007	0.042	0.015	0.051	0.017	0.030	0.023	0.034	0.038	0.017	0.016	0.032
stress = 0.05	0.007	0.046	0.017	0.055	0.018	0.032	0.026	0.038	0.041	0.018	0.017	0.034
stress = 0.01	0.009	0.054	0.019	0.063	0.021	0.037	0.032	0.047	0.047	0.021	0.020	0.040
stress = 0.001	0.010	0.067	0.023	0.075	0.025	0.044	0.042	0.062	0.056	0.025	0.023	0.047
Basel II	0.150	0.150	0.040	0.040	0.103	0.094	0.039	0.105	0.065	0.083	0.105	0.105

Parameter Estimates - Point in Time

Parameter	$\hat{\alpha}$	$\hat{\alpha}$ SE	$\hat{\beta}$	$\hat{\beta}$ SE	AC	$\hat{\omega}$	$\hat{\omega}$ SE
HLC	-2.362	0.070	13.873	4.233	0.003	0.050	0.008
HLO	-2.758	0.055	39.604	4.515	0.004	0.066	0.011
BAC	-1.975	0.090	5.591	2.169	0.007	0.086	0.012
NCR	-2.292	0.074	13.083	2.337	0.014	0.118	0.017
AMD	-2.392	0.070	15.351	4.092	0.006	0.080	0.011
AMI	-2.274	0.095	10.818	4.464	0.015	0.121	0.017
EDU	-1.699	0.138	3.342	1.710	0.009	0.094	0.018
MAF	-2.642	0.098	28.516	5.669	0.006	0.075	0.015
MOH	-2.148	0.052	9.470	1.290	0.007	0.086	0.012
PUN	-2.247	0.085	11.218	3.149	0.007	0.082	0.012
PIM	-2.318	0.083	10.208	5.132	0.008	0.090	0.013
REV	-2.536	0.079	22.946	4.651	0.010	0.099	0.014

Results - Point in Time

	HLC	HLO	BAC	NCR	AMD	AMI	EDU	MAF	MOH	PUN	PIM	REV
Expected loss												
no stress	0.016	0.011	0.040	0.029	0.016	0.020	0.076	0.015	0.037	0.026	0.016	0.015
stress = 0.10	0.035	0.020	0.089	0.058	0.034	0.051	0.219	0.045	0.059	0.058	0.037	0.035
stress = 0.05	0.038	0.022	0.098	0.064	0.037	0.058	0.250	0.052	0.063	0.064	0.042	0.039
stress = 0.01	0.047	0.027	0.121	0.077	0.046	0.074	0.332	0.072	0.072	0.080	0.053	0.049
stress = 0.001	0.062	0.035	0.157	0.098	0.059	0.101	0.469	0.114	0.085	0.106	0.071	0.065
Value-at-Risk												
no stress	0.024	0.018	0.068	0.061	0.029	0.045	0.125	0.027	0.063	0.044	0.030	0.031
stress = 0.10	0.054	0.038	0.158	0.135	0.066	0.124	0.362	0.087	0.111	0.107	0.077	0.078
stress = 0.05	0.060	0.042	0.175	0.149	0.073	0.140	0.410	0.102	0.119	0.119	0.087	0.087
stress = 0.01	0.075	0.053	0.215	0.182	0.091	0.180	0.525	0.143	0.139	0.149	0.111	0.111
stress = 0.001	0.100	0.071	0.278	0.234	0.119	0.244	0.693	0.226	0.169	0.197	0.151	0.149
BIl Value-at-Risk												
no stress	0.155	0.116	0.124	0.096	0.114	0.123	0.201	0.111	0.151	0.134	0.112	0.111
stress = 0.10	0.251	0.178	0.228	0.165	0.190	0.236	0.433	0.232	0.213	0.238	0.205	0.197
stress = 0.05	0.267	0.188	0.246	0.177	0.202	0.256	0.475	0.256	0.223	0.256	0.221	0.211
stress = 0.01	0.304	0.213	0.287	0.205	0.232	0.301	0.571	0.317	0.245	0.296	0.258	0.245
stress = 0.001	0.357	0.251	0.346	0.246	0.274	0.366	0.707	0.417	0.276	0.354	0.312	0.295

Asset Correlation - Point in Time

	HLC	HLO	BAC	NCR	AMD	AMI	EDU	MAF	MOH	PUN	PIM	REV
no stress	0.003	0.004	0.007	0.014	0.006	0.015	0.009	0.006	0.007	0.007	0.008	0.010
stress = 0.10	0.005	0.009	0.013	0.024	0.011	0.026	0.019	0.012	0.013	0.012	0.014	0.017
stress = 0.05	0.005	0.009	0.014	0.026	0.012	0.027	0.021	0.014	0.014	0.013	0.015	0.018
stress = 0.01	0.006	0.011	0.016	0.030	0.014	0.031	0.026	0.017	0.016	0.015	0.018	0.021
stress = 0.001	0.007	0.014	0.019	0.035	0.016	0.037	0.034	0.022	0.019	0.017	0.021	0.025
Basel II	0.150	0.150	0.040	0.040	0.104	0.095	0.039	0.106	0.066	0.083	0.105	0.106

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Conclusion

- Stress testing might be implemented to address **effects not captured by the basic risk model**
- Here: **estimation error**
- Constructing confidence intervals allows stress testing with explicitly **assigning probabilities** to the stress scenarios
- Identify sectors where stressed capital **exceeds regulatory buffers**
- Different results **depending on rating methodology**
- Approach can be **applied to any (empirical) credit risk model**
- Extensions may include **LGD, EAD, and correlations** among all variables