Reject Inference in Survival Analysis by Augmentation

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Background

Survival Models:	Increasingly used to predict specific features (e.g. profitability) of applicants individually and as groups.
Traditional Handicap:	Models calibrated only on previously accepted applicants may inadequately assess repayments prospects for future applicants.
Previous Experience:	Disproportionately higher weighting of more marginal borrowers in logistic regression and probit models does not improve performance ranking and under-mines discernment of the appropriate cut-off point (See e.g. Banasik and Crook, 2007).
New Question:	Since survival models deploy data more comprehensively to answer very specific questions, can weighting improve predictions.

Augmentation (also called Re-weighting):

Apply weights to accepted applicants to reflect the extent to which similar applicants have been rejected. Specifically, weight accepted cases by ***1/P(accepted)*** from a logistic regression model predicting acceptance.

Simple Cox Proportionate Hazard Model

Cumulative ***Hazard*** *at time* ***t***: $H(t,X) = H_0(t) * \exp(\beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p)$

Cumulative ***Survival*** *at time* ***t***: $S(t,X) = \exp[-H(t,X)]$

Failure Probability at time t: $P(F) = S(t-1,X) - S(t,X)$

Previous evidence (e.g. Banasik & al., 1999 or Stepanova & Thomas, 2003) suggests that the $\beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_p X_p$ component of this ***survival model*** performs ***comparably well*** with ***logistic regression*** prediction of default in spite of the handicap that the survival model retains the same ranking of applicants for all periods and the logistic regression model does not.

Since weighting has not helped logistic regression models, it seems that weighting-induced improvement in survival models will need to be at a level ***beyond ranking*** and depend on the contribution of the ***baseline hazard*** function.

DATA AVAILABLE FOR ANALYSIS

Whole Data Set: Details for 147179 applicants granted credit over a 40 month period between 1994 and 1997.

Loans were between £300 and £15000 (mean ≈ £2350) for terms between 6 months and 10 years (mean ≈ 26 months).

Data includes the credit score (but not the model) used to assess applicants.

Data is censored in that survival observation ceases:

- At the point of *normal* repayment
- Alternative delinquency occurs (e.g. early payment when loan default behaviour is being modelled, and vice versa),
- By the limit of the 40 month sample period
- Data for no more than 36 or 37 months for any applicant.

Working Data: Defaulters given disproportionately higher prominence to simulate a complete population of applicants with payment performance.

Applicants are ranked by credit score and banded into groups to simulate different rejection regimes. For each band non-defaulters are selected by proportional stratified random sampling.

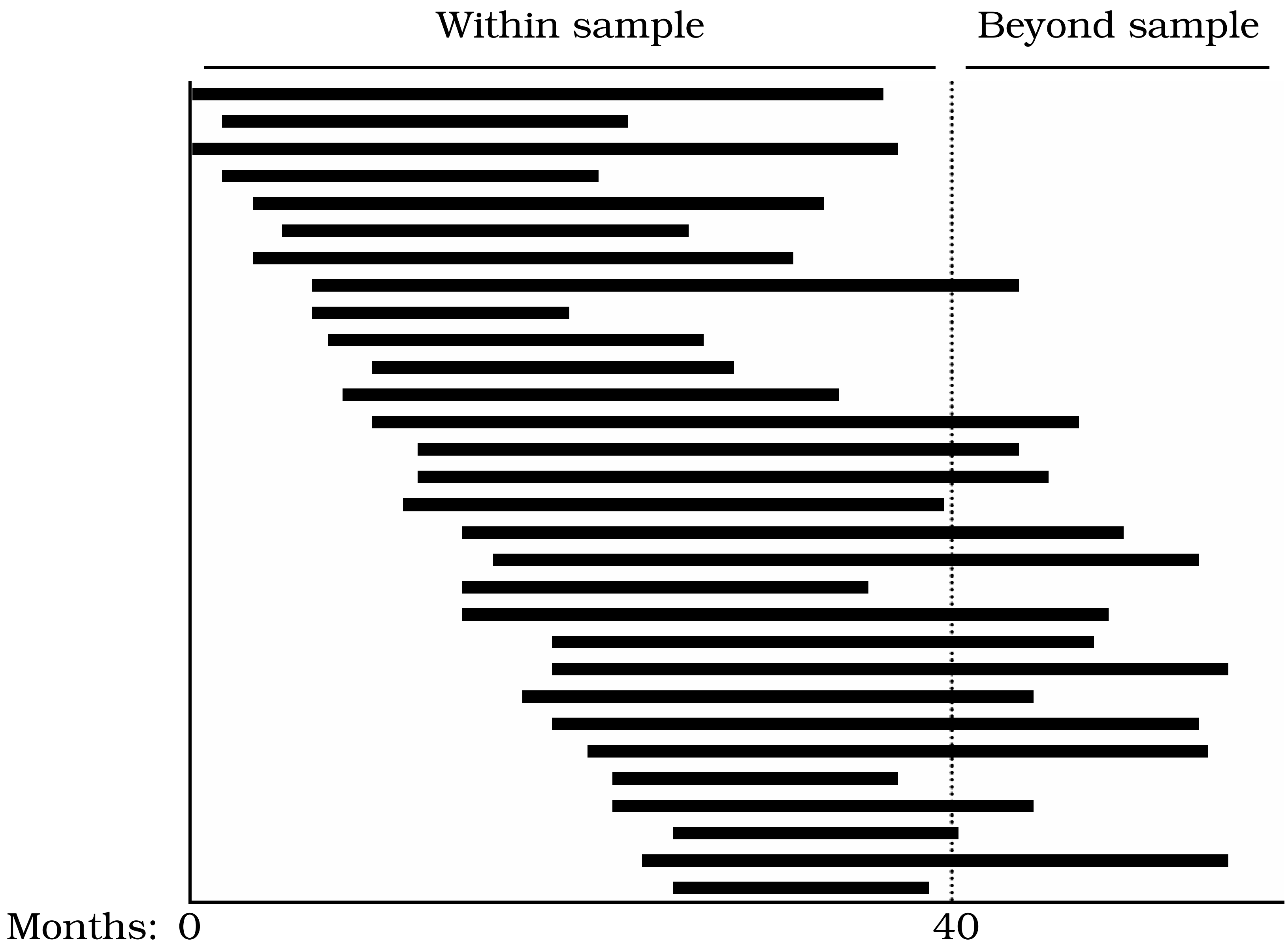


Figure 1: Illustration of Sample Observation Limits

Table 1: Worst repayment status – numbers by credit score

Credit score	Open – no arrears	Closed early	Closed normally	Short arrears	Arrears of 3+ months		Total Loans
Available sample:							
288 – 361	8310	8675	11436	1029	**215**	(.7%)	29665
263 – 287	6189	9953	10447	1852	**512**	(1.8%)	28953
242 – 262	5094	10622	10433	2598	**939**	(3.2%)	29686
220 – 241	3830	10738	9870	3632	**1532**	(5.2%)	29602
11 – 219	3139	9353	9525	4722	**2534**	(8.7%)	29273
Total	26562	49341	51711	13833	**5732**	(3.9%)	147179
Working Sample:							
288 – 361	2759	2882	3800	344	**215**	(2.2%)	10000
263 – 287	2064	3321	3486	617	**512**	(5.1%)	10000
242 – 262	1606	3348	3288	819	**939**	(9.4%)	10000
220 – 241	1155	3240	2978	1095	**1532**	(15.3%)	10000
11 – 219	878	2610	2660	1318	**2534**	(25.3%)	10000
Total	8462	15401	16212	4193	**5732**	(11.5%)	50000

Table 2: Worst repayment status – working sample numbers by band

	Open – no arrears	Closed early	Closed normally	Short arrears	Arrears of 3+ months		Total Loans
Training sample:							
Band 1	1931	2017	2660	241	**151**	(2.2%)	7000
Band 2	3376	4342	5100	673	**509**	(3.6%)	14000
Band 3	4500	6686	7402	1246	**1166**	(5.6%)	21000
Band 4	5308	8954	9487	2013	**2238**	(8.0%)	28000
Band 5	5923	10781	11348	2936	**4012**	(11.5%)	35000
Holdout sample:							
Band 1	828	865	1140	103	**64**	(2.1%)	3000
Band 2	1447	1861	2186	288	**218**	(3.6%)	6000
Band 3	1929	2865	3172	534	**500**	(5.6%)	9000
Band 4	2276	3837	4065	862	**960**	(8.0%)	12000
Band 5	2539	4620	4864	1257	**1720**	(11.5%)	15000

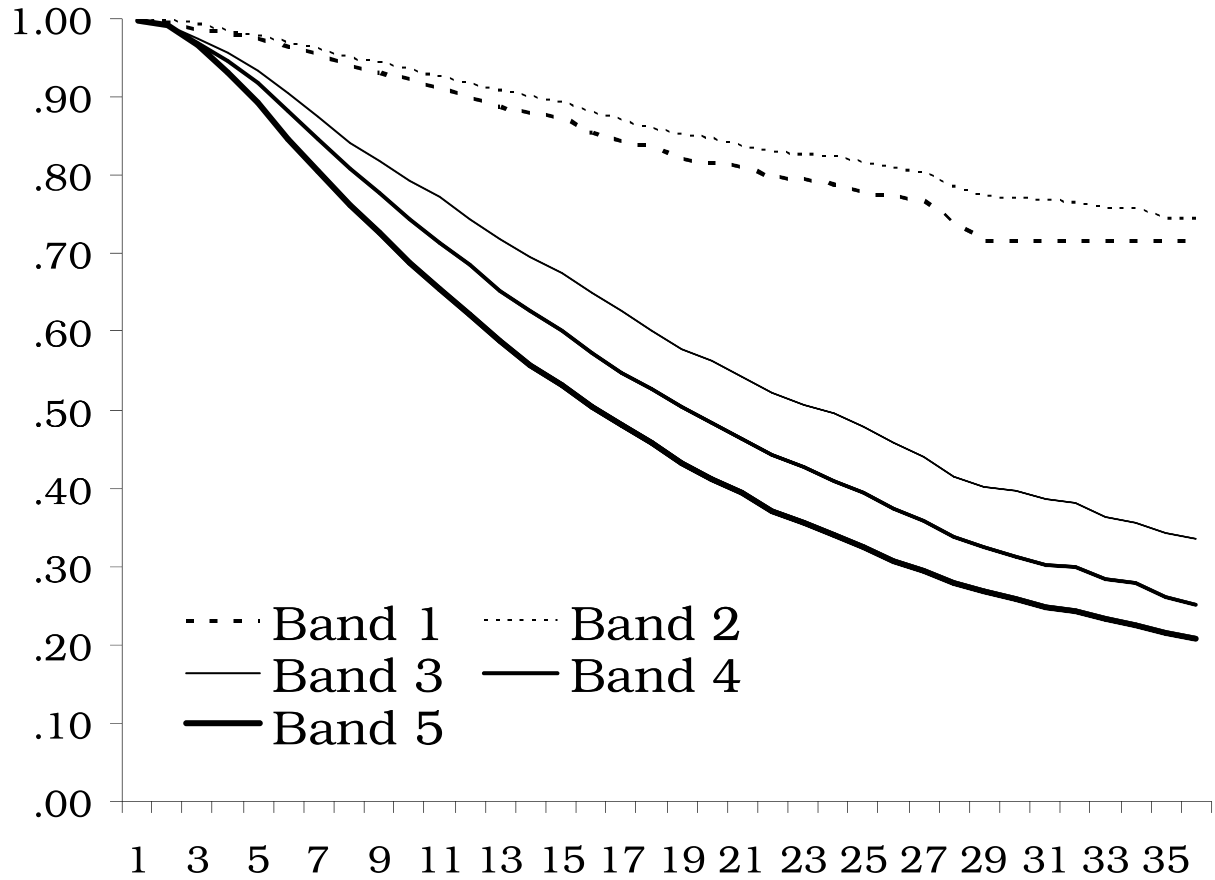
Table 3: Explanatory variables and their band 5 characteristics

	Course categories	Defaulter Function: Minimum frequency	Defaulter Function: Wald Statistic	Early Re-payer Function: Minimum frequency	Early Re-payer Function: Wald Statistic
Age at time of application	6	1271	107.79 (.000)	1271	70.98 (.000)
Loan amount	4	785	20.69 (.000)	4528	10.45 (.015)
Yr at current address	6	3733	110.77 (.000)	11564	24.00 (.000)
Yr with current employer	7	1632	390.95 (.000)	1632	19.56 (.003)
Gender	2	11756	29.16 (.000)	11756	1.27 (.260)
No. of dependent children	4	375	21.82 (.000)	375	10.89 (.012)
Frequency of pay	4	1077	232.63 (.000)	515	6.43 (.093)
Home phone no. given	2	1897	10.08 (.002)	1897	9.97 (.002)
Insurance premium	6	2010	304.15 (.000)	1348	15.44 (.009)
Loan type (single/joint)	2	9241	53.62 (.000)	9241	1.08 (.299)
Marital status	4	692	20.26 (.000)	692	22.89 (.000)
Loan term	4	836	13.89 (.003)	836	180.84 (.000)
Home ownership status	4	1977	95.64 (.000)	1977	3.35 (.340)
Loan purpose	7	960	753.74 (.000)	4447	181.97 (.000)

Table 4: Comparison of Course Classification Approaches

	Initial	Bivariate		Multivatiate			Rev.
	Class	Fine	Course	Course	Fine	Revised	Class
Refinance		.0000	.0000	.0000	.0000	.0000	
Account std		.0000	.0000	.0000	.0000	.0000	
Graduate loan		.0000	.0000	.0000	.0000	.0000	
Remortgages		.0000	.0000	.0000	.0000	.0000	
General living	A	-.3128	-.3988	-.4391	-.5379	-.5460	A
Redecoration	A	-.4340	-.3988	-.4391	-.3964	-.5460	A
Other vehicles	B	-.5581	-.6445	-.6902	-.7120	-.6911	B
Car repair	B	-.5843	-.6445	-.6902	-.7980	-.7714	C
Van	B	-.6103	-.6445	-.6902	-.8721	-.8868	D
Other n.e.s.	B	-.6309	-.6445	-.6902	-.6298	-.5460	A
Mixed purchases	B	-.6532	-.6445	-.6902	-.6916	-.6911	B
Other specific	C	-.7454	-.7840	-.8270	-.6860	-.6911	B
Music instrument	C	-.7541	-.7840	-.8270	-.9149	-.8868	D
Holiday	C	-.7554	-.7840	-.8270	-.9316	-.8868	D
Weddings	C	-.8108	-.7840	-.8270	-.6979	-.6911	B
Electrical	C	-.8251	-.7840	-.8270	-.9928	-1.0486	E
Furniture	C	-.8289	-.7840	-.8270	-.7671	-.7714	C
Motor Cycle	C	-.8479	-.7840	-.8270	-1.0989	-1.0486	E
Car > 3 yr old	D	-.9222	-.9225	-1.1697	-1.1829	-1.1809	F
Motor Caravan	D	-.9880	-.9225	-1.1697	-.9919	-1.0486	E
Car < 3 Yr old	E	-1.0215	-1.0366	-.9326	-1.0581	-1.0486	E
Home Improved	E	-1.0437	-1.0366	-.9326	-.8595	-.8868	D
Boat	E	-1.0518	-1.0366	-.9326	-1.0595	-1.0486	E
Kitchen units	E	-1.0663	-1.0366	-.9326	-.7258	-.6911	B
Caravan	E	-1.2021	-1.2286	-1.1082	-1.1668	-1.1809	F
New car	E	-1.2472	-1.2286	-1.1082	-1.0568	-1.0486	E
Honeymoon	E	-1.4754	-1.2286	-1.1082	-1.5991	-1.1809	F

Baseline cumulative survival function for defaulters



Baseline cumulative survival function for early payers

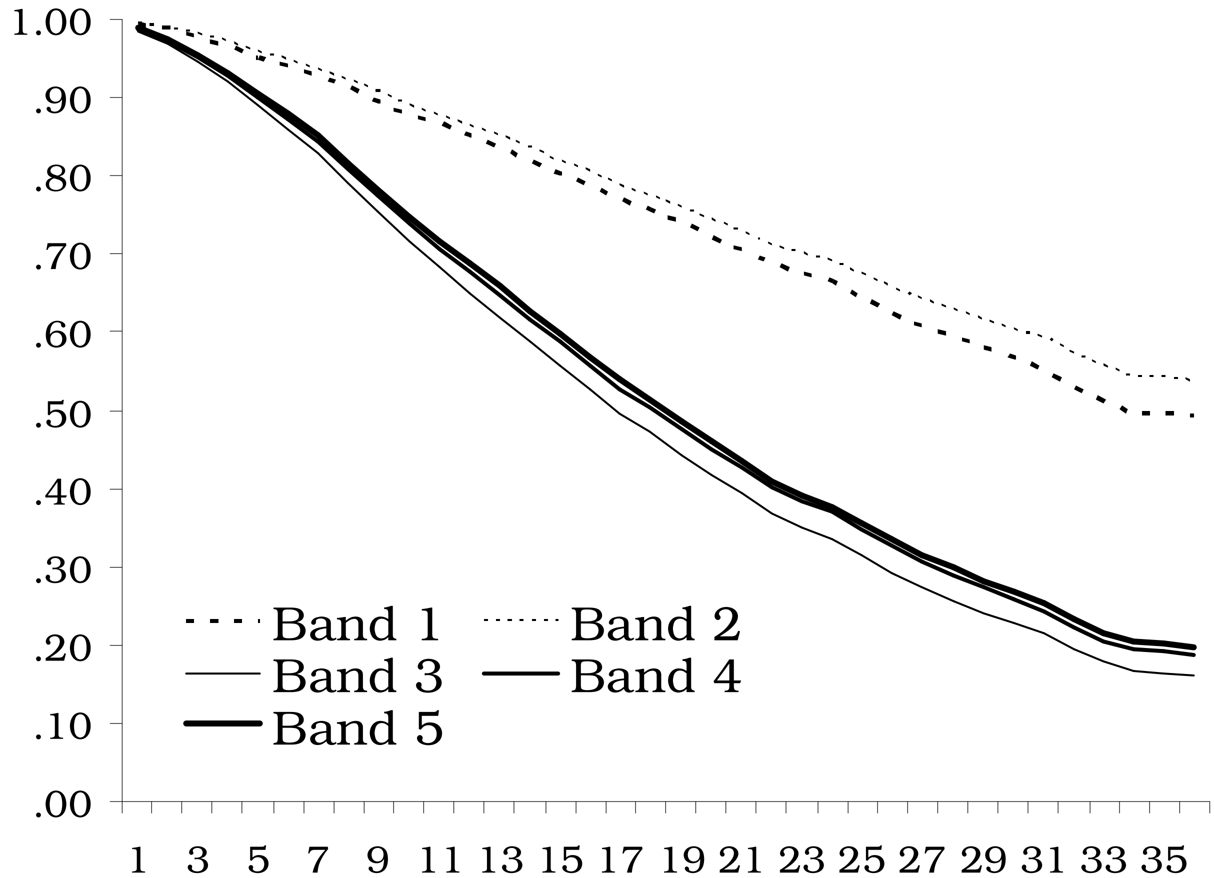


Figure 2: Baseline cumulative survival – without weighting.

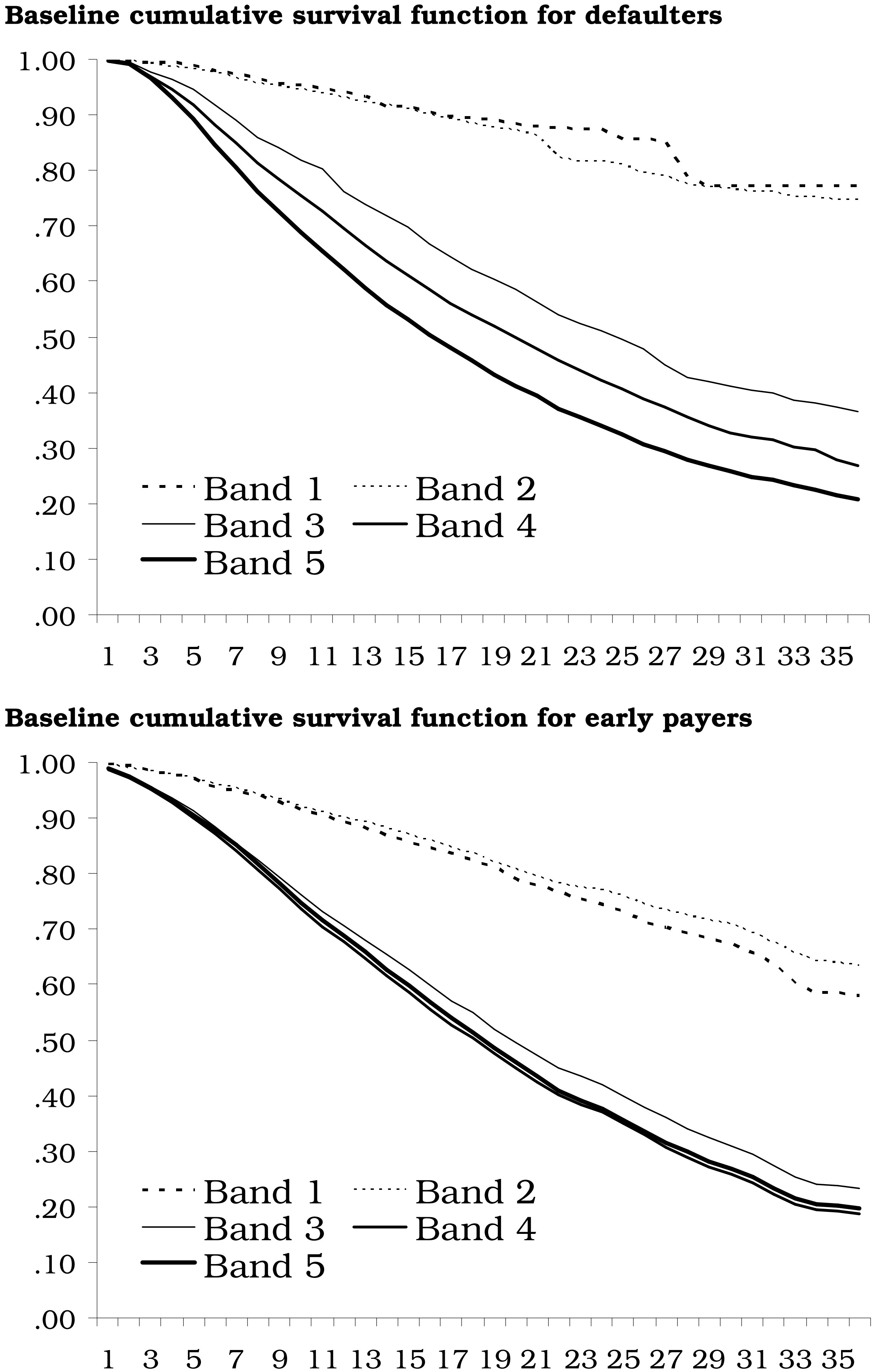


Figure 3: Baseline cumulative survival – with weighting.

Sample Calculations for Profit Potential Loss

Amount:	5000	Monthly Rate:	1.10%	Potential Receipts:	5715
Rate (AER):	14.00%	Monthly Payment:	238.12	Potential Profit:	715
Months:	24				

Loss of Potential Profit for Default at end of Month:

Month:	**1**	**2**	**3**	**4**	**5**	**6**	…	**24**	**25**	**26**
	5715	5715	5715	5715	5715	5715	…	5715	5715	5715
Receipts:	0	0	0	238	476	714	…	5001	5239	5477
Lost Potential:	**5715**	**5715**	**5715**	**5477**	**5239**	**5001**	**…**	**714**	**476**	**238**

Loss of Potential Profit for Default at end of Month:

	1	**2**	**3**	**4**	**5**	**6**	**…**	**21**	**22**	**23**
Opening Balance:	5000	4817	4632	4444	4255	4064	…	927	699	468
Interest	55	53	51	49	47	45	…	10	8	5
Receipts:	–238	–238	–238	–238	–238	–238	…	–238	–238	–238
Closing Balance:	4817	4632	4444	4255	4064	3870	…	699	468	236
	5715	5715	5715	5715	5715	5715	…	5715	5715	5715
Repayment:	5055	4870	4682	4493	4302	4108	…	937	707	474
Penalty:	111	107	103	99	94	90	…	21	16	10
Prior Payments:	0	238	476	714	952	1191	…	4762	5001	5239
Receipts:	5166	5215	5261	5306	5349	5389	…	5720	5723	5723
Lost Potential:	549	500	454	409	366	326	…	–5	–8	–8

Table 5: Profit Expectations Using Own Band Parameter Estimates

		Expected loss of profit potential				
		Loan default		Early repayment		
	Potential profit	Within sample	Beyond sample	Within sample	Beyond sample	Expected profit
Training cases:						
Band 1	4 651 176	543 997	48 286	632 167	63 764	3 362 962
Band 2	8 691 092	1 903 890	210 147	1 234 765	113 920	5 228 370
Band 3	12 316 771	4 206 323	492 695	1 754 059	151 914	5 711 780
Band 4	15 397 040	7 204 094	774 638	2 172 204	187 929	5 058 175
Band 5	17 888 737	11 112 989	916 310	2 467 231	203 047	3 189 160
Holdout cases:						
Band 1	1 974 259	243 029	21 907	261 991	23 783	1 423 549
Band 2	3 682 176	844 951	91 518	516 650	44 605	2 184 452
Band 3	5 231 653	1 820 593	206 847	747 310	60 911	2 395 992
Band 4	6 504 010	3 106 851	329 601	921 189	73 700	2 072 669
Band 5	7 530 934	4 759 390	384 344	1 037 206	79 582	1 270 412

Table 6: Expected and observed profit potential losses within sample

	Loan Default		Early payment		Combined		Total
	Expected	Observed	Expected	Observed	Expected	Observed	Error
Training cases:							
Band 1	543 997	487 633	632 167	576 758	1 176 164	1 064 391	10.50%
Band 2	1 903 890	1 636 611	1 234 765	1 102 536	3 138 655	2 739 147	14.59%
Band 3	4 206 323	3 598 588	1 754 059	1 558 263	5 960 382	5 156 851	15.58%
Band 4	7 204 094	6 094 446	2 172 204	1 901 570	9 376 298	7 996 016	17.26%
Band 5	11 112 989	9 454 253	2 467 231	2 115 240	13 580 220	11 569 493	17.38%
Holdout cases:							
Band 1	243 029	228 081	261 991	231 984	505 020	460 065	9.77%
Band 2	844 951	747 830	516 650	448 025	1 361 601	1 195 855	13.86%
Band 3	1 820 593	1 645 964	747 310	627 966	2 567 903	2 273 930	12.93%
Band 4	3 106 851	2 639 726	921 189	776 488	4 028 040	3 416 214	17.91%
Band 5	4 759 390	3 979 541	1 037 206	860 987	5 796 596	4 840 528	19.75%

Table 7: Band 5 (holdout cases) profit expectations using each band's estimates

		Expected loss of profit potential				
		Loan default		Early repayment		
	Potential profit	Within sample	Beyond sample	Within sample	Beyond sample	Expected profit
Unweighted estimation:						
Band 1	7 530 934	2 070 232	163 194	1 000 867	84 861	4 211 780
Band 2	7 530 934	2 625 419	246 828	1 084 334	83 552	3 490 801
Band 3	7 530 934	3 489 000	358 569	1 082 265	81 773	2 519 327
Band 4	7 530 934	4 078 344	399 910	1 063 414	81 664	1 907 602
Band 5	7 530 934	4 759 390	384 344	1 037 206	79 582	1 270 412
Weighted estimation:						
Band 1	7 530 934	1 640 457	219 782	894 741	87 233	4 688 721
Band 2	7 530 934	2 621 331	266 937	1 096 549	87 673	3 458 444
Band 3	7 530 934	3 436 104	341 864	1 068 034	79 968	2 604 964
Band 4	7 530 934	4 022 388	391 692	1 055 673	80 452	1 980 729
Band 5	7 530 934	4 759 390	384 344	1 037 206	79 582	1 270 412

Table 8: Band 5 (holdout case) potential losses with each band's estimates

	Loan Default		Early payment		Combined		Total
	Expected	Observed	Expected	Observed	Expected	Observed	Error
Unweighted estimation:							
Band 1	2 070 232	3 979 541	1 000 867	860 987	3 071 099	4 840 528	–36.55%
Band 2	2 625 419	3 979 541	1 084 334	860 987	3 709 753	4 840 528	–23.36%
Band 3	3 489 000	3 979 541	1 082 265	860 987	4 571 265	4 840 528	–5.56%
Band 4	4 078 344	3 979 541	1 063 414	860 987	5 141 758	4 840 528	6.22%
Band 5	4 759 390	3 979 541	1 037 206	860 987	5 796 596	4 840 528	19.75%
Weighted estimation:							
Band 1	1 640 457	3 979 541	894 741	860 987	2 535 198	4 840 528	–47.63%
Band 2	2 621 331	3 979 541	1 096 549	860 987	3 717 880	4 840 528	–23.19%
Band 3	3 436 104	3 979 541	1 068 034	860 987	4 504 138	4 840 528	–6.95%
Band 4	4 022 388	3 979 541	1 055 673	860 987	5 078 061	4 840 528	4.91%
Band 5	4 759 390	3 979 541	1 037 206	860 987	5 796 596	4 840 528	19.75%

Table 9: Profit implications of applicants wrongly expected unprofitable

	Potential profit	Observed potential lost within sample		Expected potential lost beyond sample		Neglected Profit by Selection
		Default	Early	Default	Early	
Unweighted estimation:						
Band 1	315 065	338	50 992	25 231	2 114	236 390
Band 2	497 475	1 536	76 087	42 875	3 418	373 559
Band 3	877 593	2 159	134 997	69 211	5 929	665 297
Band 4	1 190 337	2 521	173 280	86 310	7 867	920 359
Band 5	1 466 689	2 521	206 518	97 811	9 217	1 150 622
Weighted estimation:						
Band 1	221 522	0	33 641	11 985	1 234	174 662
Band 2	562 934	1 898	82 525	46 547	3 971	427 993
Band 3	892 403	414	139 556	72 244	7 111	673 078
Band 4	1 164 621	2 236	169 903	85 348	7 968	899 166
Band 5	1 466 689	2 521	206 518	97 811	9 217	1 150 622

Table 10: Net profit gain from avoiding negative expected profits

	Correctly expected unprofitable				
	Potential Profits	Observed default loss within sample	Profit Saved by Selection	Neglected Profit by Selection	Net Gain from Selection
Unweighted estimation:					
Band 1	159 055	614 457	455 402	236 390	219 012
Band 2	267 284	1 021 182	753 898	373 559	380 339
Band 3	431 817	1 630 221	1 198 404	665 297	533 107
Band 4	524 625	2 026 424	1 501 799	920 359	581 440
Band 5	584 731	2 284 436	1 699 705	1 150 622	549 083
Weighted estimation:					
Band 1	98 966	378 660	279 694	174 662	105 032
Band 2	281 441	1 058 395	776 954	427 993	348 961
Band 3	413 508	1 555 598	1 142 090	673 078	469 012
Band 4	506 148	1 960 719	1 454 571	899 166	555 405
Band 5	584 731	2 284 436	1 699 705	1 150 622	549 083

Conclusions

There does exist a considerable **profitability perception problem** where parameters are estimated in the context of a **high rejection rate**, but **not** otherwise.

Attempting to correct this problem by **augmentation** as the form of reject inference **makes matters worse**. Perception and hence applicant selection are undermined.

> Because weighting has no perceptible influence on the baseline hazard function, the features that made augmentation ineffective for logistic regression probably also pertain here:
>
> Weighting effectively narrows the range of observed attributes and performance considered and thereby undermines efficiency unless there is a corresponding benefit from better focus. Note that:
>
> - Accepted cases have relatively homogeneous performance; the relationship between behaviour and P(Accepted) is weak
> - Focus is very narrowly placed on relatively few rejectable accepted cases, so the loss of efficiency is great.