Contributions towards a Theoretical Framework for the Efficient Use of Multiple Scorecards

Peter Beling, Kanshukan Rajaratnam, George Overstreet

University of Virginia

30 August 2007

Motivating Scenario

- Suppose we have scores from a set of score cards, each predicting the same outcome (e.g. Good/Bad).
- We might choose to use the outputs from the best model, and discard the rest.
- That strategy results in a potential loss of diversity of information, since each model may:
  - employ a different subset of observations,
  - consider different variables,
  - make different assumptions about the relationship among the variables and use different design concepts.
- By making use of both scorecards it may be possible to do better than we could with either alone.
  - Bates and Granger [1] illustrate that the linear combination of multiple forecasts can have lower forecast error than any of the individual forecasts.
  - Tumer and Ghosh [2] show the average combination of multiple classifiers will generate a lower misclassification error rate than each of the individual classifiers.

Combining Scorecards

- Suppose scorecards S and T are available and we have a development population such that for we know each individual:
  - an outcome (e.g., Good/Bad)
  - scores s and t, from S and T, respectively.
- In practice, many lenders construct two-dimensional cutoff policies (e.g., swapping rejects under one scorecard into the accept category if the other score is sufficiently high).
- Zhu, Beling, and Overstreet [3] describe a Bayesian approach to constructing a combined scorecard that, in theory, makes optimal use of the available information.

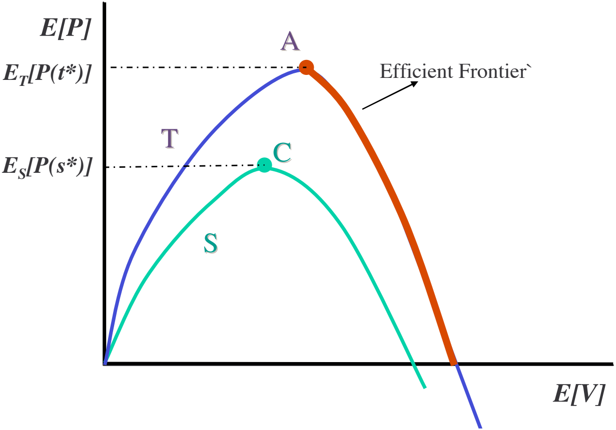
Choosing between Scorecards

- It may be that scorecard combination is not an option because a database common to the two scorecards does not exist. For example, the scorecards might have been created using one or more data elements that do not overlap in time or geography.
- Suppose scorecards S and T are available and we have two development populations. For one population we know for each individual:
  - an outcome (Good/Bad)
  - score s from S.
- For the other population we know for each individual:
  - an outcome (Good/Bad)
  - score t, from T.
- In this scenario, the baseline decision problem is to choose which scorecard to use with applicants: S or T?

Choosing Scorecards by Profit-Volume Efficiency

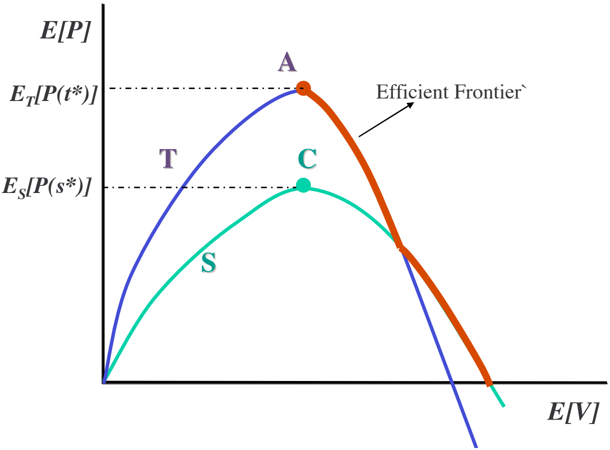
- Beling, Covaliu and Oliver [4] outline a method for choosing between scorecards.
- Which scorecard to use depends on the predictive characteristics of the scorecards and also on how one resolves tradeoffs between business objectives.
- Here we will restrict attention to the business objectives of profit, denoted by P, and volume, denoted by V.
- Setting a score cutoff s_c determines an expected profit, $E_S[P(s_c)]$, and an expected volume, $E_S[V(s_c)]$, for the portfolio if we use that cutoff rule and scorecard S for the whole applicant population. Likewise for scorecard T.
- In $E[P] - E[V]$ space we can then plot curves parameterized by score cutoff that represent the operating points that can be achieved with each scorecard. The non-dominated set of operating points is called the *efficient frontier*.

Efficient Frontier when One Scorecard is Dominant



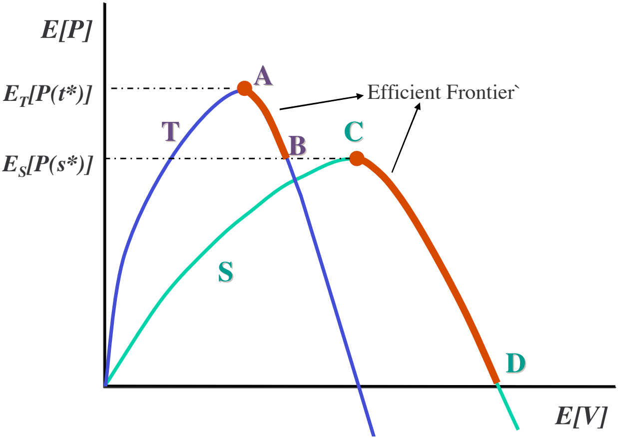
With one dominant scorecard, the efficient frontier is determined by the dominant scorecard.

Continuous Efficient Frontier when Neither Scorecard is Dominant



With no dominant scorecard, the efficient frontier is determined by both scorecards.

Discontinuous Efficient Frontier when Neither Scorecard is Dominant

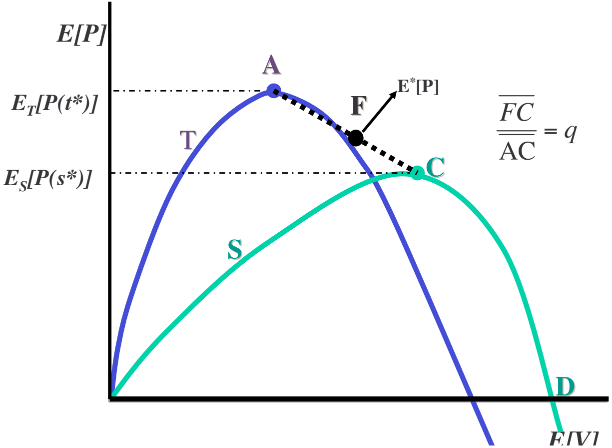


The efficient frontier is discontinuous, and consists of $[A, B) \cup [C, D]$ on the red curve.

Fixed Allocation: A Simple Randomization Scheme

- What if instead of deterministically choosing between S and T as outlined by Beling, Covaliu and Oliver [4], we randomize scorecard selection?
- This strategy, like any strategy that makes use of both scorecards, would only make sense if there is no dominant scorecard.
- Assume that we have a coin that lands heads with probability q. Then one way to randomize is to flip the coin for each applicant and score that individual with T if the coin is heads and with S if the coin is tails.
- An accept-reject decision is then made by applying a score cutoff s_c to those applicants scored by S and a score cutoff t_c to those applicants scored by T.
- In expectation, then, a fraction q of the population is allocated to scorecard T and a fraction $(1 - q)$ is allocated to S. We call this the *fixed allocation* case.

The Max E[P] point

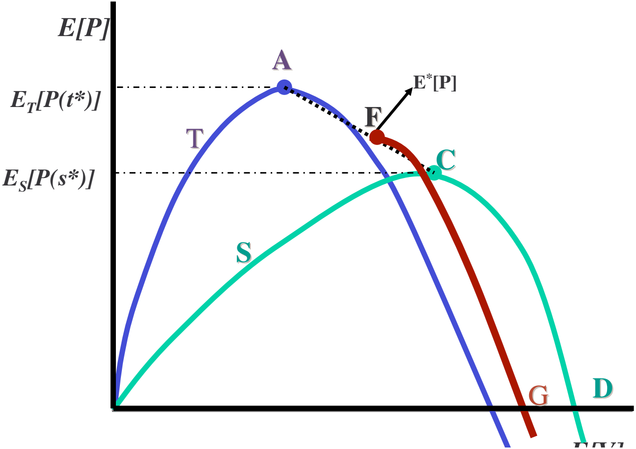


Draw a chord from point A to C. The maximum $E[P]$ point under our randomized strategy is found by moving from C toward A a fraction q of the total chord length.

The Efficient Frontier for Fixed Allocation

- It is interesting to consider what the efficient frontier looks like under randomization as we have defined it so far, flipping a coin that has a fixed probability q of heads.
- The only controls that are available for choosing an operating point are the score cutoffs s_c and t_c for scorecards S and T, respectively.
- Consider operating at the maximum expected profit point for fixed allocation. If we would like to operate with a higher expected volume, then we will have to lower the cutoff on S or T.
- Lower the cutoff of the scorecard that gives the smallest decrease in profit per incremental increase in volume.

The Efficient Frontier for Fixed Allocation

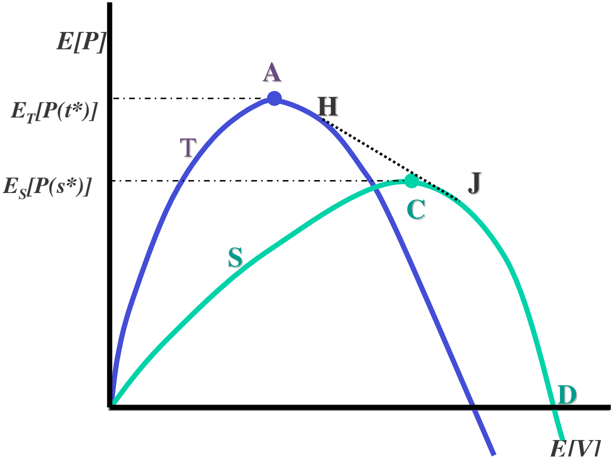


Efficient frontier is shown in red.

Variable Allocation: A More Complex Randomization Scheme

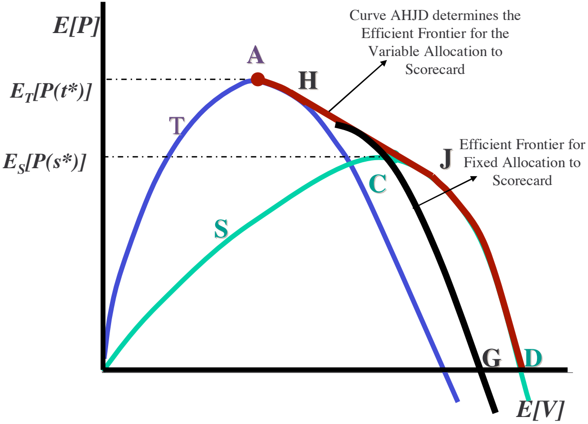
- Suppose now that the decision maker has the option to choose the probability, q, that the coin comes up heads.
- The best q will depend on where the decision maker intends to operate in $E[P]$-$E[V]$ space.
- In expectation, a fraction q of the population is allocated to scorecard T and a fraction $(1-q)$ is allocated to S. We call this the *variable allocation* case because the decision maker has control over the expected allocation of applicants to each scorecard.
- The efficient frontier for variable allocation constructed by imagining a line that is tangent to the efficient frontiers of both S and T.

Efficient Frontier for Variable Allocation



- A-H-J-D is the efficient frontier.
- Points on A-H and J-D are determined by scorecards T and S, respectively.
- Points on H-J are determined by both scorecards T and S.

Efficiency Comparison for Fixed and Variable Allocation



The efficient frontier for variable allocation dominates that for fixed allocation.

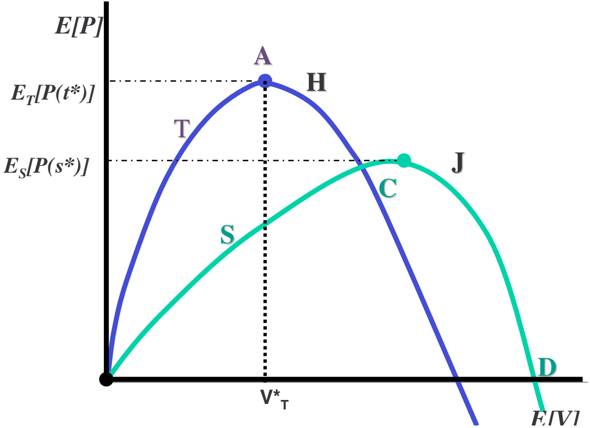
Adverse Action Considerations

- Thus far, we have described randomization as the decision maker flipping a coin for each applicant, as part of the scoring process.
- A difficulty may arise if we consider two applicants who have identical historical credit data, as one of the two may be rejected while the other is accepted.
- What adverse action reasons could a lender provide in such a case? There seems to be a fairness issue here.
- Instead of flipping a coin for each applicant, the lender could flip once to determine which scorecard be used for the entire pool of applicants.
- This results in the same efficient frontier one gets when flipping a coin for each applicant. However, applicants with exactly the same historical credit data will either be all rejected or be all accepted.

Consideration of Profit Variance

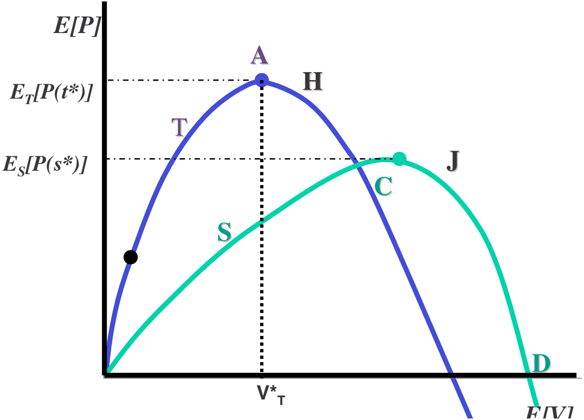
- Thus far we have considered only expectation when thinking about the profit associated with a portfolio.
- The probabilistic nature of the scoring means that there is variance in portfolio profit.
- Profit variance increases as one moves along the efficient frontier to lower cutoffs.
- Randomization over scorecards introduces yet another source of profit variance.
- To see the overall effects, we consider the efficient frontier for a given tolerance on profit variance.

Efficient Frontier given a Tolerance for Variance



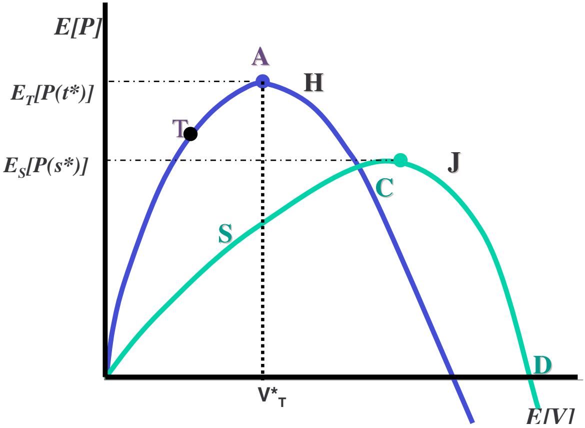
With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance



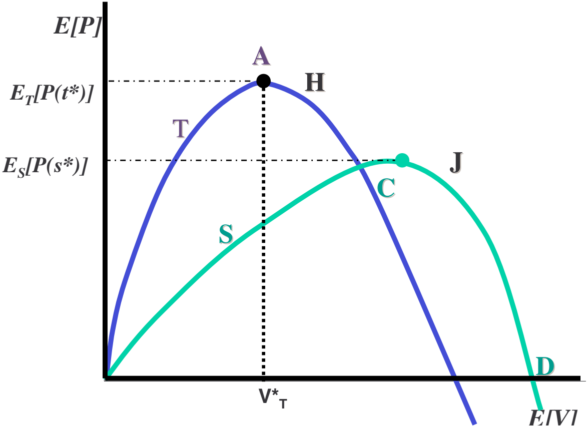
With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance



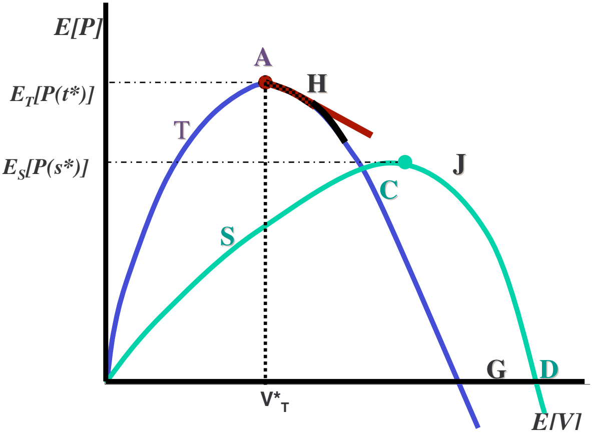
With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance

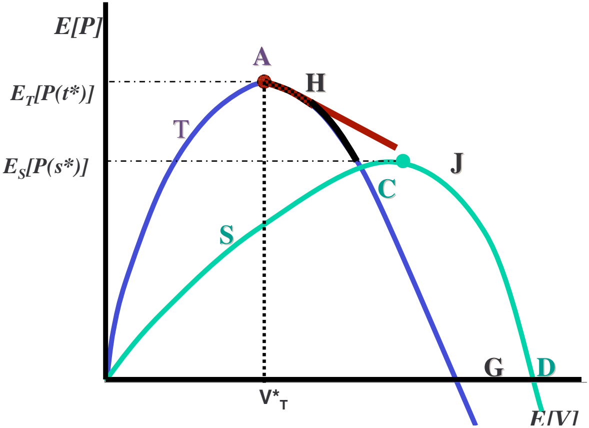


With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance

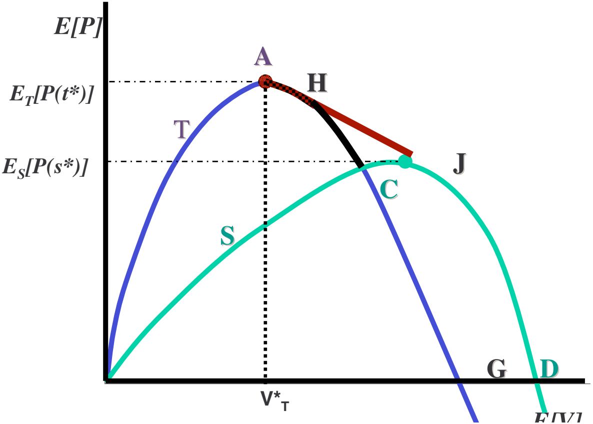


With randomization in red, without randomization in black.



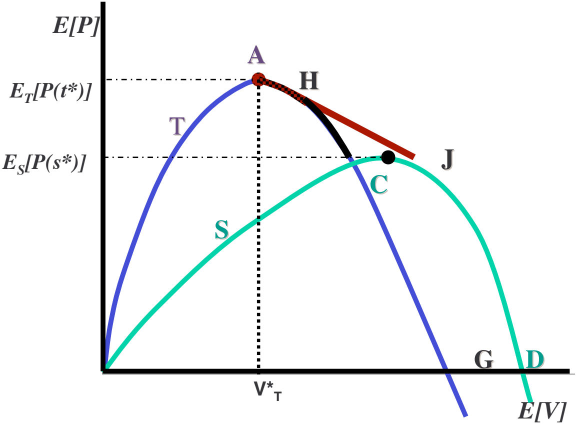
With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance



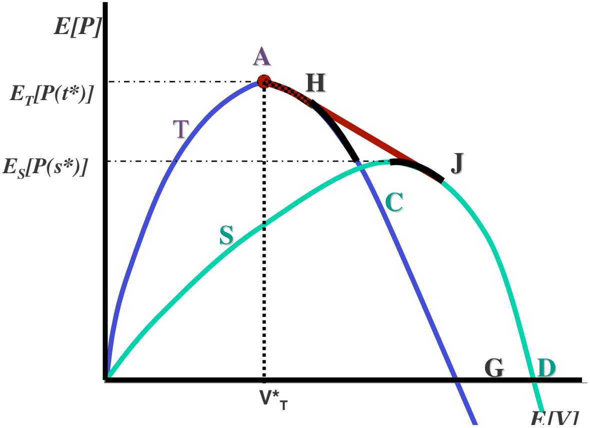
With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance



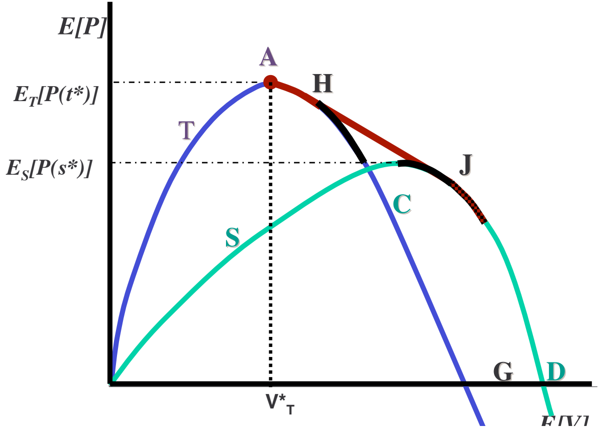
With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance



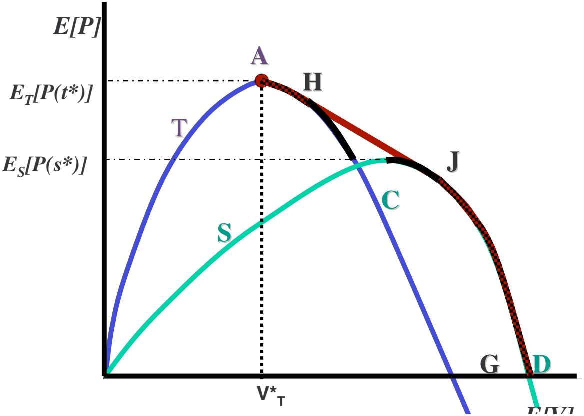
With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance



With randomization in red, without randomization in black.

Efficient Frontier given a Tolerance for Variance

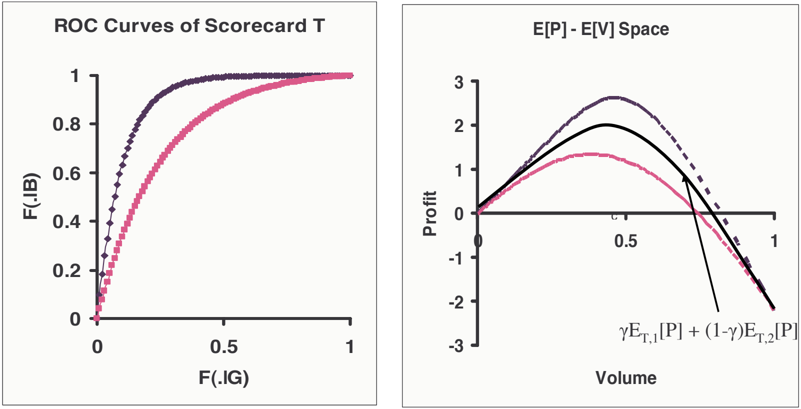


With randomization in red, without randomization in black.

Implications for Scoring Decisions given Economic Forecasts

- Consider a situation where the decision maker has access to two scorecards, one that dominates in one economic scenario while the other dominates in the other economic scenario.
- Thomas [5] mentions the use of two scorecards, one for good economic conditions and another for bad economic conditions.
- Example, if there is a .75 probability of realizing economic scenario 1, it would be intuitive to score, 75% of the applicant population using the scorecard that is dominant in economic scenario 1 and 25% of the population on the other scorecard. However, this would result in an inefficient portfolio.
- Let γ be the probability of economic scenario 1.

Scorecard T in two Economic Scenarios

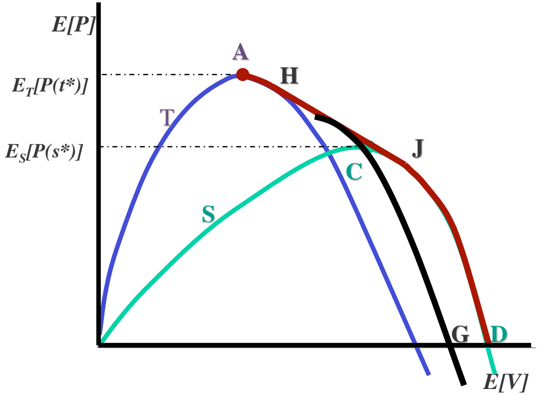


Given a score cutoff S_c, we obtain one $E[V(S_c)]$ in both scenario for the each scorecard (i.e. T) and we can determine the expected profit $E_T[P(S_c)] = \gamma E_{T,1}[P(S_c)] + (1-\gamma)E_{T,2}[P(S_c)]$.

Economic Scenarios

- Similarly, for scorecard S, we can determine $E_S[P(S_c)] = \gamma E_{S,1}[P(S_c)] + (1-\gamma)E_{S,2}[P(S_c)]$
- We saw through an earlier example, that given γ, the probability of realizing economic scenario 1, it would be intuitive to score, γ of the applicant population using dominant scorecard in that respective scenario and $(1-\gamma)$ of the population for the other scenario.
- However, the above will result in an inefficient portfolio as it resembles fixed allocation to scorecards (as seen earlier).

Variable Allocation to Economic Scenarios



- A-H-J-D determines the efficient frontier for variable allocation to economic scenarios.
- Fixed allocation to economic scenarios will result in an inefficient portfolio.

Final Remarks

- Flat Maximum effect suggests that a way to get improvement is not to refine algorithms and statistical methodologies on a given set of data but rather through new sources of data [6], [7].
- Given that data is the path to improvement, there exists situations where there is no common datasets from which to construct a single dominant scorecard. This is precisely the situation we have addressed.
- Our findings:
  - Randomization of scorecards results in a superior efficient frontier, and we know how to construct that frontier;
  - There is little need to be concerned with variance, if you were not worried about it before;
  - Our thought experiments for randomized scorecards have natural application to segmentation strategies and to scoring decisions given forecasts of future economic behavior.

References

1. Bates JM and Granger CWJ (1969). The Combination of Forecasts. *OR* 20: 451–468.
2. Tumer K and Ghosh J (1996). Classier Combining: Analytical Results and Implications. *Proceedings of the AAAI-96 Workshop on Integrating Multiple Learned Models for Improving and Scaling Machine Learning Algorithms*, Portland, Or: 126–132.
3. Zhu H, Beling P and Overstreet G (2001). A study in the combination of consumer credit scores. *J. Opl Res Soc* 52: 974–980.
4. Beling P, Covaliu Z and Oliver RM (2005). Optimal Scoring cutoff policies and efficient frontiers. *J. Opl Res Soc* 56: 1016–1029.
5. Thomas LC, (2000). A survey of credit and behavioral scoring: forecasting financial risk of lending to consumers. *Int J. Forecast.* 16: 149–172.
6. Overstreet GA, Bradley EL and Kemp RS (1992). The flat-maximum effect and generic linear scoring models: a test. *IMA Journal of Applied in Business and Industry* 4: 97–109.
7. Crook JN, Beling P and Overstreet G (2000). Technological Development in Credit Scoring: A Strategic Review. *Invited paper at Equifax Symposium on Credit*, Leeds University, October 2000.