

Cyclical adjustment of Point-in-Time (PiT) PD

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1 Introduction

1.1 The Problem

The CRD stipulates that to calculate its capital requirements a bank should use “long-run average PD’s”. Here, “long-run” should be taken as at least sufficiently long to cover a full credit cycle. Not all banks, however, have sufficiently reliable data on which to train a credit scoring model for such a long period. This is because to train a model, detailed information on the values of all explanatory variables are needed. Thus the problem arises, how to construct a model that predicts not short term PD, reflecting the current credit environment, but the required long-run PD.

One approach to resolving this problem, is to model the credit cycle separately, and then adjust or calibrate the estimated short term model to reflect the long-term average. The reason this may be simpler often banks have good data regarding the dependent variable alone, i.e. defaults or losses, even when the detailed application or behavioral data describing the obligors is unavailable equally far back in time.

1.2 Compliance issues

Although some supervisors may not have decided to whether such an approach is acceptable or not, the FSA, for one, has addressed the subject in a detailed memorandum.¹ In the memorandum, the approach outlined above is referred to as the “scalar” or “variable scaling factor” methodology. The stated purpose of the memorandum is to “inform the industry how the FSA intends to react to such approaches in the IRB waiver application process, including how it will interpret the resulting impact upon the stress testing requirements” (p. 1) The memorandum proceeds to discuss the concept of “rating philosophy” i.e. the way cyclical is reflected in ratings, and concludes that “...the FSA’s approach is to be liberal as regards firms’ choice of the rating philosophy, but to expect the consequence of their decisions to be reflected in the stress testing process which looks at how the IRB requirement might rise in a “once in 25 years” downturn.” (p.3)

As regards the “scalar methodology”, the FSA acknowledges that: “The effect of this mechanism is to convert the original rating system into one with Through-the-Cycle (TtC) characteristics in terms of the stability with the cycle of the resulting IRB capital requirements” (p.4). It voices its concern, however, that “a number of firms proposing to use such scalar approaches have not given sufficient thought to how [its variation over time] will operate.”

In conclusion, the FSA affirms that “it is acceptable in principle for UK firms to use methodologies of [this] type”, while presenting two important caveats. One is that “ a firm must be able to overcome the considerable conceptual and technical challenges involved”. (p.4) The supervisor warns that it will take a skeptical questioning approach to firms’ proposals in this regard. The other caveat is that notwithstanding the adjustment for “normal” cyclical, a firm’s stress testing must still include a “once in 25 years” stress test based on the parameters of the underlying rating system, in addition to the stress factor built into the cyclical adjustment used in the Pillar 1 capital calculation.

In particular, the FSA voices concern that cyclical adjustment may not distinguish between portfolio degradation that is due to cyclical fluctuation, and bank specific, or idiosyncratic credit

¹ FSA: Use of long run Probabilities of Default, counter-cyclical scaling factors, and the interaction of these with economic cycle stress testing. Memorandum to the Credit Risk Standing Group, 30 October 2006. (FSA Memorandum 2006)

deterioration. If such a distinction is not made, the capital requirements of a bank may become insensitive to changes in a bank's specific risk profile, e.g. due to a riskier acquisition strategy, undermining the purpose of the IRB approach. Other sources of concern include the use of industry data and proxies, the length of the time series used to determine the economic cycle, and the appropriateness of the scaling factor to all portfolios which it is used to scale

Finally, the FSA noted that due to scaling, the result of the standard Pillar 1 stress test (BIPRU 4.3.38) will tend to zero, and the capital requirement will remain unchanged regardless of where we are in the economic cycle. This is why an additional "once in 25 years" stress test is mandatory. On a positive note, however, the FSA acknowledges that adequate modeling of the credit cycle for the purposes of scaling to the long-run-average may be helpful in constructing the worst case scenario and provide support to appropriate stress testing.

1.3 Overview of the approach

In this paper, a model of the credit cycle is introduced, that has the potential to fulfill the regulatory demands an implementation of the variable scaling factor methodology must meet. The model is based on a structural time-series analysis of credit losses incurred in the retail and small business portfolio of a commercial bank. A structural approach means that no attempt is made to explain the credit cycle, or to causally relate it to other observed elements, only to determine its stochastic structure, with a view to establish dynamics. The emphasis is placed on the prediction of future realizations of the time-series, based on information from past occurrences only. This is appropriate, given its cyclical character of the series, which is adopted as a research hypothesis and confirmed by the data. A corollary is that the process must be monitored with respect to the possibility of exogenous structural change, but this would in fact also hold true for any other type of model.

The structural model is cast in a state-space form incorporating the basic economic features of the credit cycle, i.e. a cyclical component and a long-term trend. With reference to the principle of parsimony, no further systematic elements are introduced to the model. The time-varying parameters of the model are then estimated by the application of the Kalman filter.² Using the methodology advocated by Harvey (1989), *inter alia*, the so-called *hyperparameters* governing the dynamics of the system are estimated from historical data using the method of maximum likelihood, to train or tune the filter using the whole historical series.

Due to the Kalman-filter setup, the model can be continually updated as new data arrives, and an optimal balance is achieved at each time between the weight of historical elements and new evidence.

2 Data

2.1 Definition of dependent variable

The dependent (and only) variable in the Credit Cycle model is the series of incurred losses or write-offs, covering the period 1990-2004, and the data was retrieved from the bank's files and from historical annual reports early in the year 2005.

Economic growth, and to some extent inflation, characterize this period and impose a stochastic trend that tends to obscure the parameter of interest. The data is in monetary terms, and to detrend the loss-series, it is transformed into percentage terms by dividing each observation by the average size of the commercial lending balance in the previous year. Notwithstanding the fact

² Harvey A., *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press, 1989. Hereafter: (Harvey)

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that the bank has grown through mergers in this period, they have minimally affected both losses and the size of the commercial lending portfolio and it is safe to assume that the series provides a good measure of the development of credit loss over time.

For the most recent years (2003 and 2004) an additional adjustment must be made to account for the fact that not all write-offs that will eventually be made have yet been formally entered into the bank's books. To adjust for this, 80% of the specific provisions are added to the losses, based on the fact that historically 80% of specific provisions have resulted in a write-off over a 10 year period.

The data is segmented into "Individuals" and "Other legal entities", meaning small corporates, qualifying for treatment as members of the retail asset class under Basel II.

3 Model methodology

3.1 Model approach

3.1.1 Timing of losses

The Credit Cycle Model is a structural time series model that utilizes a Kalman filter algorithm to estimate the credit cycle based on a state-space representation of The bank's historically incurred losses. Definitive losses, in the sense of accounting write-offs, are booked at an arbitrary point in time often many years after the corresponding default. Thus the time of write-off is of no help in constructing the credit-cycle. By using the time-stamp of the corresponding default event (as defined in the Basel II guidelines), this problem can be resolved. This simple adjustment transforms the write-off data into a time series that may be interpreted as a progression of "consequential" or "serious" defaults.

3.1.2 Meaningful losses

The series of consequential defaults is admittedly not consistent with the series of simultaneously occurring defaults at any given time because it disregards the defaults that are "cured" without any recorded loss. We may argue, however, that this series is more meaningful for the present purpose because it includes credit events if and only if they are economically meaningful in a clear sense of this term. In another perspective, the series is simply a series of loss events with a meaningful temporal structure.

3.1.3 The Kalman filter

A major advantage of the Kalman-filter algorithm is that information is optimally discounted as time advances instead of arbitrarily discarding older data at a given cut-off point, and giving all other data the same weight regardless of its time of occurrence, as ordinary ARMA model regression would. Thus, the Kalman filter picks up changes in the underlying data process quickly, and adapts to changes more readily than other common approaches.

The Kalman filter addresses the general problem of trying to estimate the state of a given process x that is governed by a stochastic differential equation (SDE):

$$x_k = Ax_{k-1} + Qw_{k-1}$$

which is observed through another process z governed by

$$z_k = Hx_k + Rv_k$$

where w_k and v_k are standard normal variables. In this context we can think of x as being the state of the credit cycle and z as the observed losses.

The Kalman filter algorithm provides the solution to the above SDE, proceeding in the following way:

1. Project the state ahead: $\hat{x}_k^- = A\hat{x}_{k-1}^-$
2. Project the error covariance ahead: $P_k^- = AP_{k-1}^-A^T + Q$
3. Compute the Kalman gain: $K_k = P_k^-H^T(HP_k^-H^T + R)^{-1}$
4. Update the estimate with the new measurement z_k : $\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$
5. Update the error covariance: $P_k = (I - K_kH)P_k^-$
6. Go to 1.

We assume that the credit cycle is a stationary periodic process with zero drift where the matrices A , Q and H take the form:

$$A = \begin{bmatrix} \eta & 0 & 0 & 0 \\ 0 & \xi & 0 & 0 \\ 0 & 0 & \kappa & 0 \\ 0 & 0 & 0 & \kappa \end{bmatrix}, Q = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \rho \cos(\omega) & \rho \sin(\omega) \\ 0 & 0 & -\rho \sin(\omega) & \rho \cos(\omega) \end{bmatrix} \text{ and } H = [1 \ 0 \ 1 \ 0]$$

where the estimates of η, ξ, κ, ρ and ω are obtained by minimizing

$$\Gamma = \sum_{k=2}^N \log(HP_k^-H^T + R) + (N-1) \log \left(\sum_{k=2}^N \frac{(z_k - H\hat{x}_k^-)^2}{(HP_k^-H^T + R)(N-1)} \right)$$

and N is the number of observation-periods. This is equivalent to the assumption that the credit cycle consists of three components: a cyclical component that behaves more or less like a sinus wave, a relatively stable long-term trend or average component, and finally a random error component or noise term. Of these, the two former define the regular structure of the component and define the state-equation, i.e. the process x above. Whether the state varies more or less between periods depends of the size of the term that drives the variance of the state, i.e. the random vector w_k .

In addition, the predictive performance of the model depends on the size of the measurement error, or noise component represented by the scalar v_k . The relationship between their variances is called the signal-to-noise ratio. If the two variance terms are normally distributed, the Kalman filter is the best available estimator of the state-space model. If they are not, it is the best *linear* estimator. On the assumption that this quantity is known, it is easy to demonstrate that the Kalman filter is the optimal – or minimum mean square error (MSE) – estimator of the unobserved components of a structural time series model. In practice, though, the various signal-to-noise ratios, or *hyperparameters*, in the model are not known and must be estimated. This is the normal

situation in applied statistics, and the estimator remains the best feasible one.³

3.2 The resulting model

Application of the Kalman filter to the data yields estimates of the various components of the structural credit cycle at each data point.

Figure illustrates the estimation results for household, or “retail individual” obligors. The figure shows the result of application of the filter to the problem of *prediction*, in the sense that at each point only information on past data points is used to obtain the estimate. Lower MSE estimates could obviously be obtained by the application of the filter to the *smoothing* problem. This is, however, considered less relevant to the purpose of estimation in this case, which is not analysis of the past, but first and foremost a prediction of the future. Figure illustrates the estimation results for losses due to legal entities other than individuals, i.e. corporates.

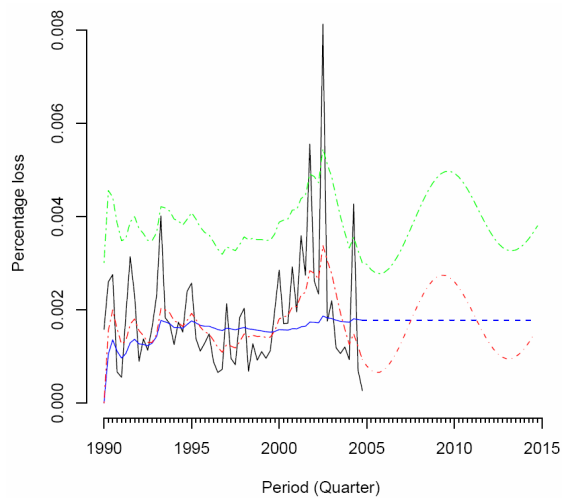


Figure 1: Kalman filter estimate of the Credit Cycle based on household-losses.

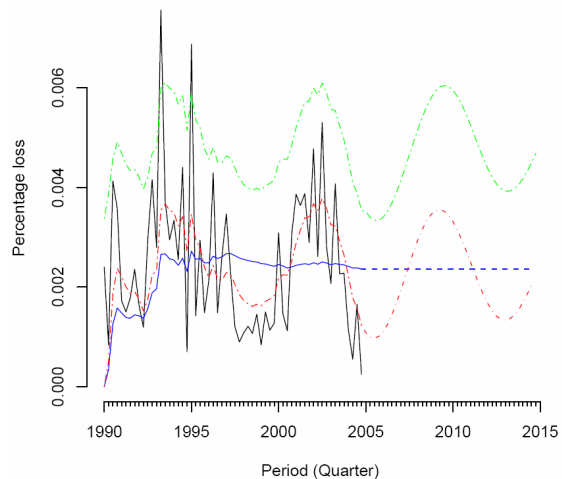


Figure 2: Kalman filter estimate of the Credit Cycle based on losses from legal entities other than individuals.

³ A practical discussion of signal extraction and the Kalman filter can be found in Harvey, p.227ff.

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The figures above show that the Kalman filter successfully approximates bank losses for the past 15 years for the retail portfolio. This period includes two recession periods that can easily be accommodated with economic analyses. The first recession represents difficulties that arose in the economy due to the transitional difficulties of the early nineties, with peaks in 1993 to 1995. During these years, the Icelandic government simultaneously fought inflation by structural measures and loosened its grip of the economy by extensive privatization and liberalization of financial markets. The second recession occurred in 2001, when a bubble in asset prices burst and the ISK was subsequently floated, resulting in a massive devaluation.

The mid-nineties recession was particularly difficult for VSE companies, as the structural changes in the economy reduced the demand for their services in many cases and partly made their way of management obsolete. Households suffered less severe losses during the mid-nineties than VSE companies but there were still significant signs of recession among households as a consequence of the problems of the corporate sector. The peak in credit losses coincides with an unusually high unemployment rate.

In 2001 many individuals had entered the financial markets and were burned by unexpected volatility when both the ISK and the stocks of many companies fell sharply. Although the 2001 setback was not as pronounced as the earlier one in the "real" sector, and even benefited the export industries, the losses of banks were no less real than earlier, and continued to have a ripple effect throughout 2002.

It may be concluded that the Credit Cycle model correctly and consistently represents the credit cycle of the Icelandic economy over the years 1990 to 2004, based on 60 quarterly intra-bank loss-data points.

3.3 Transforming the results of a PiT PD model to TTC PD

It is a prominent characteristic of the economic cycle, that credit losses in general and PD in particular tend to increase in times of recession and decrease during expansionary periods. Retail PD Model and Retail SME PD Model used by the bank in question are logistic regression models, based on data that covers a one-year period. Consequently the models pick up the effects of the economic cycle reflected in each dataset. As a result, they predict a lower or higher average PD across clients than the long-term average, depending on the state of the economy. To avoid unnecessary fluctuations in regulatory capital, PD estimates are supposed to reflect a "long-run average", i.e. an average PD including both economic expansion and economic recession. To resolve this discrepancy, banks may adjust their estimates of these risk drivers.

To address this issue, a structural time-series model was used to decompose historical losses into a stochastic cyclical component on one hand, and a fixed long-term average level on the other, in addition to a purely random element attributed to measurement error. This model was fitted on a quarterly series of "serious defaults", i.e. defaults that subsequently led to loss in 1990-2004. This was done separately for private clients and corporations. The estimation results are summarized in

Figure and Figure for the Personal and Incorporated loan portfolios respectively.

The Credit Cycle model predicts the level of serious credit events in this sense as a wave with a varying shape based on the available historical observations at each time. With the help of the predictive formula, each PiT PD estimate that is produced by the PD models can be scaled back to its estimated long-run-average. Thus, large scale migrations across rating classes and fluctuations in regulatory capital over time can be avoided, while the distinction between creditworthy and less creditworthy clients is preserved in the cross-section of the portfolio.

Assuming that the Credit Cycle model yields an accurate description of the credit cycle and that the average PD of the portfolio is a function of the credit cycle, a correction term for a logistic

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regression model can be derived. It is assumed that the average PD of the portfolio is of the form

$$PD_t = f(x_t)PD$$

where $x_t = (\mu + \eta_t)/\mu$ is an adjustment term due to the Credit Cycle, f is a polynomial of the 1st degree and PD is the TtC average PD. Now note that the predicted PD from the logistic regression models is based on a 12 month observation period. This leads to the following equation:

$$E_t^s PD_r = PDf(E_t^s x_r)$$

where E_t^s denotes the expected value operator on the time interval between t and s , $E_t^s PD_r$ is the average default rate during the observation period and $f(E_t^s x_r)$ is the average scaling of the TtC PD.

Armed with this estimate of the adjustment factor $a_t = E_t^s x_r$ it is possible to estimate f by the means of a linear regression and from that estimate the distance to the long-term average PD at any given point in time from PD_t . This permits the adjustment of the PiT PD estimate of retail clients according to the level of their long term default probability by simply dividing PD_t by $f(a_t)$. Similarly, future PD's can be estimated by the appropriate adjustment.

3.3.1 Correlation between PD and LGD

The above formulation assumes that the PD is independent of the LGD. If we say that the losses are of the form

$$L_t = PD \times (1 + \varepsilon_t) \times LGD \times (1 + r\varepsilon_t)$$

where r is the correlation between PD and LGD we see that the scaling a_t is exact for $r=0$ but for $r>0$ it is conservative during expansion and optimistic during recession. Negative correlation is implausible, and it is a general consensus that PD and LGD are positively correlated. However during recession the square root of a_t is a conservative estimate of the scaling since by assuming that $r=1$ the estimate for the loss curve reduces to

$$L_t = PD \times (1 + \varepsilon_t)^2 \times LGD$$

A more conservative estimate of the scaling would be of the form

$$A_t = \begin{cases} a_t, & a_t < 1 \\ \sqrt{a_t}, & a_t \geq 1 \end{cases}$$

And as before a new estimate for the TTC PD is obtained by dividing the PIT PD with $f(A_t)$.

3.3.2 Adjusting a logistic regression to a new mean

Given a logistic regression model, the predicted mean is adjusted in the following way. Let p_0 denote the current mean and p_1 the desired mean and let $\log(p_i/(1-p_i)) = \alpha_i + C_i, i \in (0,1)$ be the logit transformation of p_i , α_i the constant of the linear model with predicted mean p_i and C_i the contribution of the model input parameters to the mean. By assuming that the ordering of obligors by the regression model is fixed throughout the cycle, $C_0 = C_1$, we get:

$$\log(p_1/(1-p_1)) = \log(p_1/(1-p_1)) + (\alpha_0 + C_0 - (\log(p_0/(1-p_0))))$$

which gives:

$$\alpha_1 = \alpha_0 + \log(p_1/(1-p_1)) - (\log(p_0/(1-p_0)))$$

and from this we derive the correction term for a logistic regression:

$$\psi = \log(p_1/(1-p_1)) - (\log(p_0/(1-p_0)))$$

3.3.3 Portfolio degradation

In the FSA guidelines of 30 October 2006⁴ it is noted that a counter-cyclical scaling approach such as the one described here should properly address the issue of portfolio changes, with respects to credit quality, which is independent of the credit cycle. To counter the effects of these changes on the scaling, a two step approach is used. Firstly, when an estimate of the credit cycle has been obtained the scaling is applied to the PD models assuming that, on the time interval between the time of the model and when the model is re-fitted, any change in the portfolio is independent of the credit cycle. This implies that any deterioration/ improvement of the portfolio is perceived as a shift in the portfolios dynamics rather than stationary economic fluctuations. This approach is valid since the credit cycle will not change significantly during the prediction period and therefore the TtC estimate holds.

Furthermore, when the PD models are re-calibrated annually, the changes in the composition of the portfolio with regards to changes in the default rate are studied.

By internally comparing the PD of new obligors to the previous obligors' PD and externally comparing the bank's default rates to default rates published the Central Bank of Iceland, the changes due to re-calibration are determined. If the deviation in the internal obligors default rate is substantially different from previous years, the credit cycle scaling is adjusted accordingly.

4 Model development tests

This chapter describes the development test used for the Credit Cycle Model. One should note, however, that a prior restriction is made on the model since it is assumed that there is a credit cycle and it is a stationary period process. This means that the model might not be the one that best fits the data but the best model that fits the data subject to the conceptual restrictions. Therefore, conventional statistical tests may reflect poorly on model performance.

⁴ FSA: Use of long run Probabilities of Default, counter-cyclical scaling factors, and the interaction of these with economic cycle stress testing. Memorandum to the Credit Risk Standing Group, 30 October 2006.

4.1 Development testing

The tests for the Credit Cycle Model fall into two categories, one for the fit of the model to the loss series and the other for the applicability of the scaling to adjust the prediction of the logistic regression model to the long term average. To test the model's goodness of fit, the usual

R^2 metric is used, which is in this case a metric that measures the reduction in deviance with respect to a model using only a constant term.⁵ This goodness of fit test of the Credit Cycle model to the loss yielded the result 33.3% within-sample, which is a reasonable fit given the restrictions of the model.

To test the appropriateness of the scaling, monthly defaults from 2002 to present were regressed on the Credit Cycle adjustment factor. The results, which can be seen from figure 3, show that the model fits the data quite well, with an R^2 of 87%. Figure 3 illustrates this relationship.

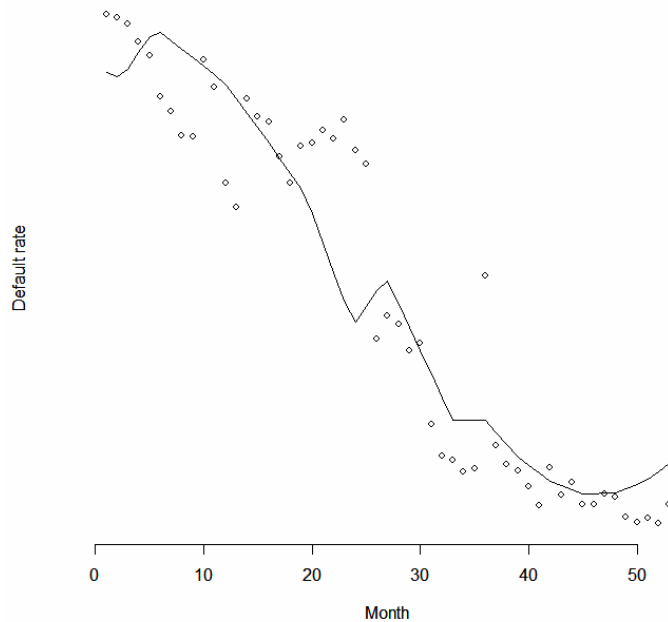


Figure 3: The observed default frequency of non-defaulted individuals within 12 months (dots) compared to the smoothed Credit Cycle adjustment term $f(A_t)$ (line).

4.2 Implications

The main purpose of the Credit Cycle model is to provide means to calibrate a PiT PD model to an arbitrary point in the Credit Cycle and as such it seems to perform adequately for the banks retail clients.

4.3 Discussion

This paper addresses the problem of obtaining TtC or long-run average PD for the purpose of calculating capital requirements when statistical models are estimated on short-run data and thus

⁵ This measure is discussed in Harvey (1989), p. 268.

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produce short-run or PiT predicted PD. It was proposed to model the credit cycle, predict the deviation from the long-run average for the prediction period, and adjust the PiT PD by a scaling factor based on this deviation in percentage terms. The scaling is applied on the level of the logistic regression to obtain a new prediction for each individual client under the restriction that the average corresponds to the long-run average term of the structural credit cycle model. The approach is conceptually simple and technically manageable. It offers optimal use of information under weak assumptions and requires only routine monitoring and maintenance. In short, it is a practical solution for a bank that possesses loss data over a full credit cycle (or two) and good obligor data a few years back, but lacks detailed data on obligor risk drivers for a sufficiently long period to qualify as “long-run”. However, the acid test is whether such an application is likely to find favor with the supervisory authority.

Section 1.2 highlights some elements of the position of the UK FSA *vis-à-vis* this methodology. Any comparison of this particular application to the FSA criteria is likely to remain hypothetical, but may still be of some interest.

Their first concern is that banks may underestimate the modeling effort needed to account for the true dynamics of the credit cycle. For its level of complexity, we hold that this model reasonably accounts for the cycle, and correctly represents the stylized concept of credit cycle that underlies the Basel II guidelines.

Secondly, they advocate a “once in 25 year” stress test as a necessary complement to cyclical adjustment of Pillar 1 PD estimates. In fact the present approach greatly facilitates the application of such a stress test for the portfolios in question, as the credit cycle model and the underlying data can be helpful in reasonably determining the level of such a stress test for the portfolios in question.

The third issue we noted concerned the ability of the scaling approach to distinguish between systematic and bank-specific portfolio deterioration. In Section 3.3.3 we proposed a mechanism to monitor the effect of the two components by regular checks. Such a mechanism could probably also be incorporated into the automated adjustment process using a model that decomposes deterioration into systematic and the idiosyncratic components based on certain criteria. But this would be at the expense of simplicity and clarity.

The remaining issues of Section 1.2 are those of industry or proxy data, the length of the time-series, and its applicability to the portfolio being scaled. None of these arise in the present application, as internal data covering two credit crunches is available and can be decomposed into reasonably similar portfolios. This does of course not mean that these data issues can be solved for all banks and portfolios, but it is likely that in many cases external data can be used to construct such a model if this is done with sufficient care.

The main weakness of the credit cycle model – and thereby of the whole adjustment approach – is one it shares with most macroeconomic models. It lies in the possibility that a structural change may radically alter the character of the credit cycle. An advantage of the Kalman-filter approach is that it is responsive to changes, especially if uncertainty rises in the system. But it is a linear mechanism and it can not accommodate sudden jumps or drops. If the timing of the break is known, however, this can be addressed by adding a dummy variable to the model. Interestingly, in 2004, a boom in real-estate prices and partial privatization of the mortgage market in Iceland combined to induce a sharp drop in default rates, as a large portion of households refinanced their debt at lower rates and longer term with mortgages. The effects are slow to appear in write-offs, but it is an interesting task to study the effect of this during the past 10 quarters using specific provisions to determine whether the credit cycle has changed markedly with this change in market structure.