Best Practice in Model Risk Quantification

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Best Practice in Model Risk Quantification

- What is Model Risk and why quantify it?
- Basic approaches to quantify model risk in practice.
- More sophisticated approaches - where do models live?
- Principles for Model Risk quantification.

The risk of using a model

Every bank decision entails risk; and

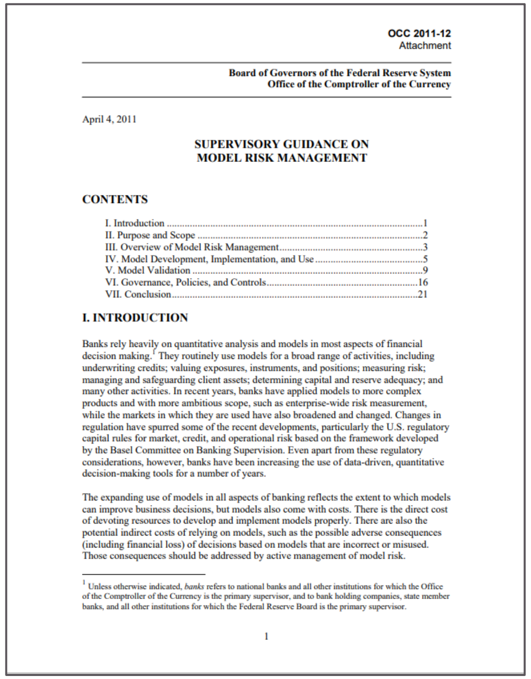
Almost every decision in a bank is determined, supported or suggested by models.

Model Risk – the risk of using a model to help make a decision.

- Operational Risk – *The Model as an algorithm, system, user-operated tool*
- Specification Risk – *The Model as an expression of policy, expert knowledge and data experience*
- Credit Risk – *The Portfolio of Models*
- Market Risk – *The Model as a commodity in a free market*

“Model Risk should be managed like other types of risk”

Supervisory Guidance on Model Risk Management : OCC Bulletin 2011-12 (= FRB SR11-7)



OCC 2011-12
Attachment

Board of Governors of the Federal Reserve System
Office of the Comptroller of the Currency

April 4, 2011

SUPERVISORY GUIDANCE ON MODEL RISK MANAGEMENT

CONTENTS

I. INTRODUCTION

Banks rely heavily on quantitative analysis and models in most aspects of financial decision making.[1] They routinely use models for a broad range of activities, including underwriting credits; valuing exposures, instruments, and positions; measuring risk; managing and safeguarding client assets; determining capital and reserve adequacy; and many other activities. In recent years, banks have applied models to more complex products and with more ambitious scope, such as enterprise-wide risk measurement, while the markets in which they are used have also broadened and changed. Changes in regulation have spurred some of the recent developments, particularly the U.S. regulatory capital rules for market, credit, and operational risk based on the framework developed by the Basel Committee on Banking Supervision. Even apart from these regulatory considerations, however, banks have been increasing the use of data-driven, quantitative decision-making tools for a number of years.

The expanding use of models in all aspects of banking reflects the extent to which models can improve business decisions, but models also come with costs. There is the direct cost of devoting resources to develop and implement models properly. There are also the potential indirect costs of relying on models, such as the possible adverse consequences (including financial loss) of decisions based on models that are incorrect or misused. Those consequences should be addressed by active management of model risk.

[1] Unless otherwise indicated, *banks* refers to national banks and all other institutions for which the Office of the Comptroller of the Currency is the primary supervisor, and to bank holding companies, state member banks, and all other institutions for which the Federal Reserve Board is the primary supervisor.

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Why quantify model risk?

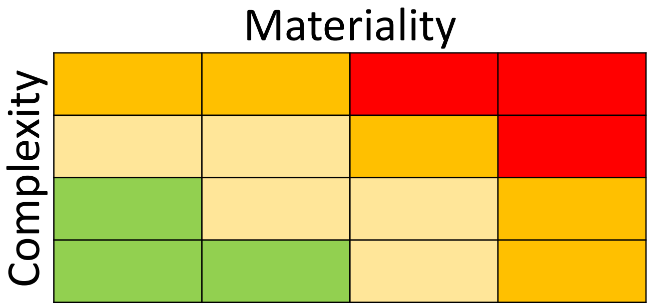
Because it helps all stakeholders

- Model Managers plan the right work priorities and resource levels
- Validators have better conversations about validation findings
- Model Users compare the risks of using a model with the benefits, and act accordingly
- The Board is comforted that models are assessed in the same scale as other principal risks
- The Regulator gains confidence that model weaknesses and errors are sized accurately and consistently, and managed in proportion to their size

Because other risks are quantified

Operational Approaches

RAGs and model status define a structural way to quantify the risk.



Validation outcome			
	5%	20%	40%
	3%	15%	30%
	1%	10%	20%
	1%	5%	10%

Model Risk impact

x% * Exposure

Linked directly with Model Management and Policy

Good for communication of risks and risk appetite

- But can we compare widely among model types, or business types?
- And how do we calibrate the numbers?
- And does it cause distorting behaviours?

Sensitivity Approaches

OCC 2011-12 *"…using a range of outputs, rather than a simple point estimate, can be a useful way to signal model uncertainty and avoid spurious precision."*

Bank of England PRA SS1/23 *"The output of the model should be compared with the outcomes of alternative theories or approaches and benchmarks, where possible."*

"…performance tests should be conducted using plausible scenarios that assess the extent to which the model can take into consideration changes…[and more detail follows]."

Conservatism:
How much model adjustment or overlay is required to make us confident that the model will continue to give the right decision?

Reverse Test:
What change in model performance or context would cause us to discontinue the model?

Scenarios and Model-shifts:
If the context changed, how differently would we build or operate the model, and how different would its output be?

Model Risk – a relative risk

Sensitivity analysis is a great framework for presenting and quantifying model risk:

- Shows directly the range of model variation under "plausible scenarios" – but we need to be systematic
- Challenger models pick out the contribution of particular weaknesses or assumptions

Its weakness is its bottom-up approach that is difficult to aggregate without over-complexity or over-simplicity

But sensitivity analysis is also the key to the next step of Model Risk:

We should assess the risk of using a model

- *versus* the risk of not using a model
- *versus* the risk of using a different model

Where do models live? – Risk Measures

A Model is a probability distribution of an outcome, conditioned on an input.

The collection of all such models is a normed linear space of distributions.

For a particular problem, the Acceptance Set is the subset of models that fit the constraints and meet required standards.

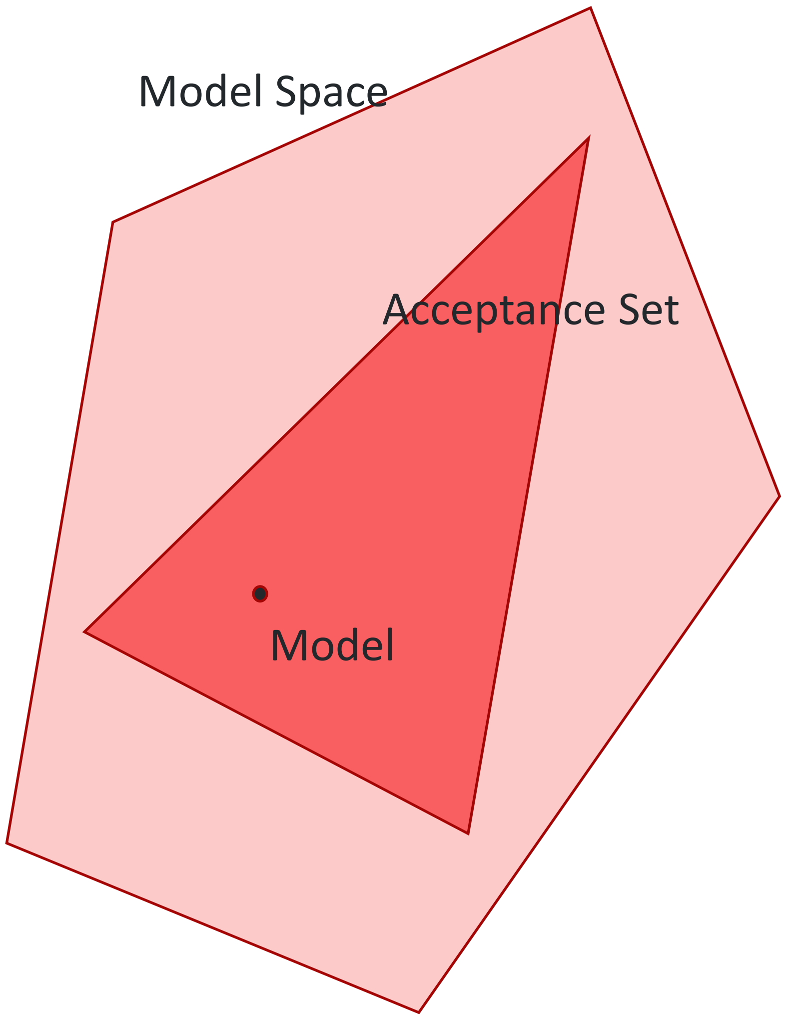
- Statistically sound, simple, optimal
- Compliant
- Meets business expectations and practical objectives
- Explainable, Fair, Ethical

The paths to the boundary describe the Model Risk of the chosen model.

- The length of the shortest path is a Risk Measure of model risk
- Convexity and other geometric properties become important and useful

This fits Model Risk quantification into the classical theory of Risk Measures such as VaR and its variants.

cf. papers by R. Cont, F. Muller, M. Righi and others.



Where do models live? – Data Spaces

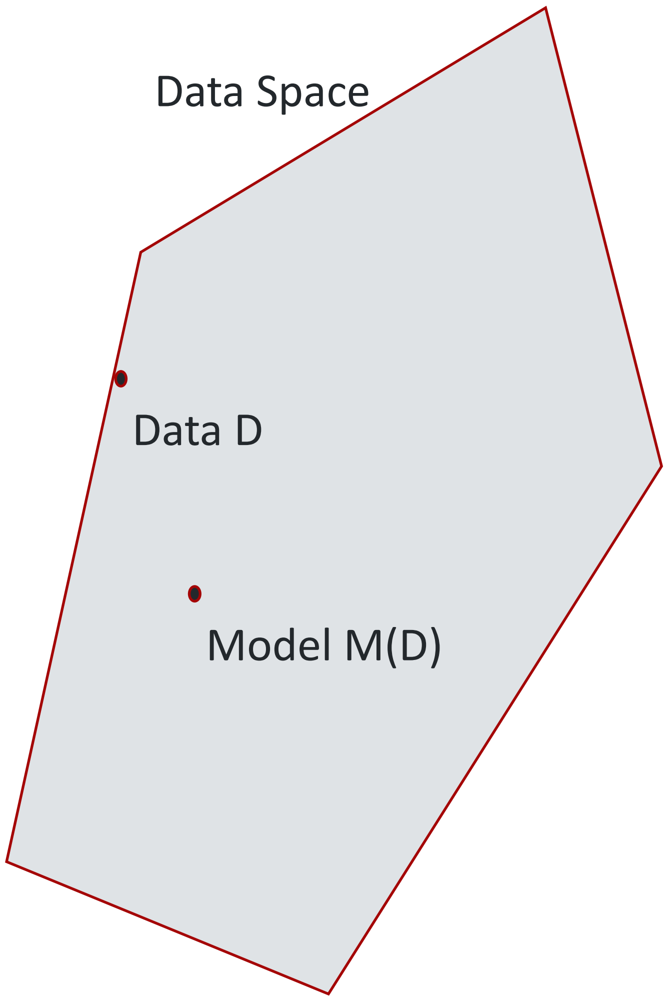
For categorical data, a dataset D can be encoded completely by Actual cell frequencies in a contingency table.

The Data Space is the set of all possible cell frequencies (normalized).

A model M built on D predicts Expected cell frequencies for the in-sample population: so a model is viewed naturally as another data point M=M(D).

Check the following (desirable / essential?) outcomes

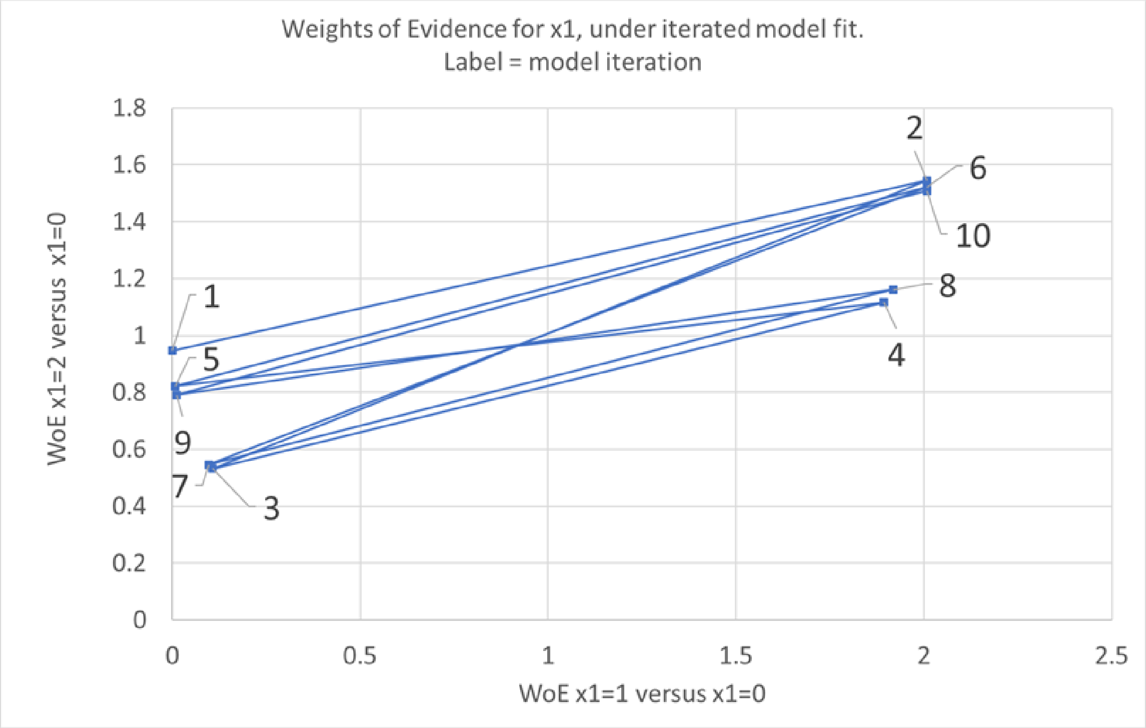
1. *M(D) has a convenient informative computable algorithmic form that allows it to be implemented as a predictor on other input populations*
2. *The modelling process D → M(D) is a continuous function*
3. *The modelling process is Iteratively Stable: $M^2(D)=M(D)$. A model built on a model's prediction results in the same model.*



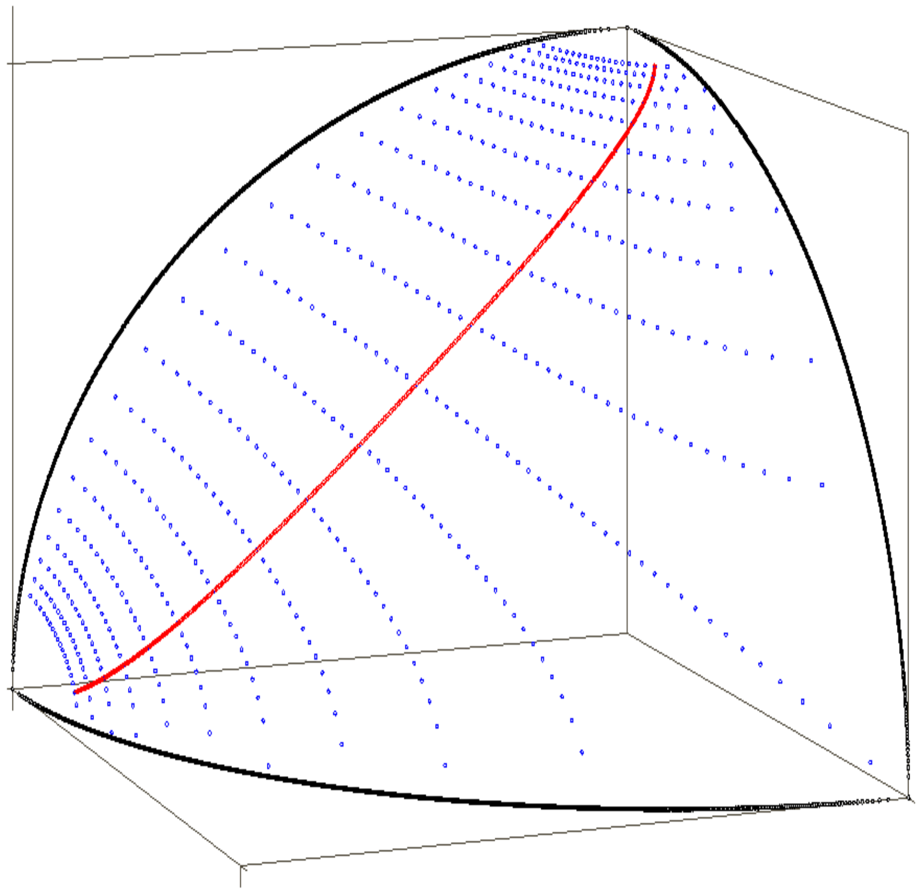
How do modelling methods check out?

- Simple regression modelling is continuous and iteratively stable
  - but factor selection process may cause discontinuities
- Decision Tree modelling is not continuous, but is iteratively stable
  - Random Forest modelling helps smooth things out
- Weights of Evidence logistic regression modelling is continuous, but not iteratively stable
  - Example: WoE applied to 3 level variable (X1)

default / total		X1 0	X1 1	X1 2
X2	0	50/1000	50/100	20/100
X2	1	5/100	5/1000	2/1000



Information Geometry – Data Spaces are curved



N. Chentsov, Category of mathematical statistics, Dokl. Acad. Nauk USSR 164 (1965), 511-514

Staying with the Data Space of contingency tables, a natural measure of diffrence is the classic Kullback-Leibler divergence. This is locally equivalent to a Riemannian metric that induces a constant positive curvature on the data space, like a sphere.

For regression, the model space is a pre-determined subset of data space from which models are selected.

Model Fitting – Maximum Likelihood Estimation (MLE) selects the point in model space that is closest (Kullback-Leibler) to the development data point.

In the diagram, blue data points contract orthogonally to their MLE fitted model in the red model space.

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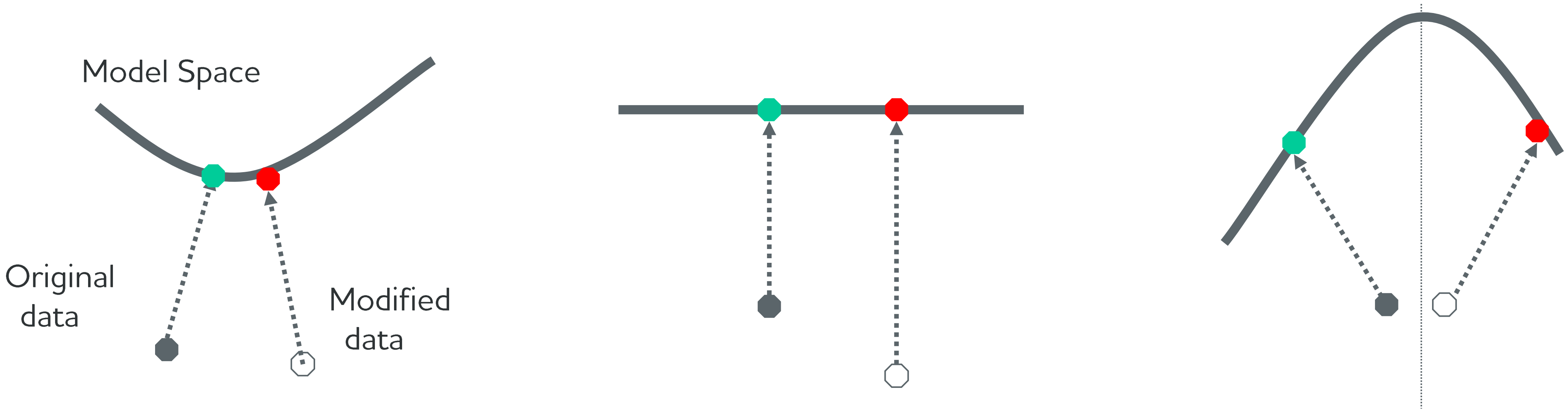
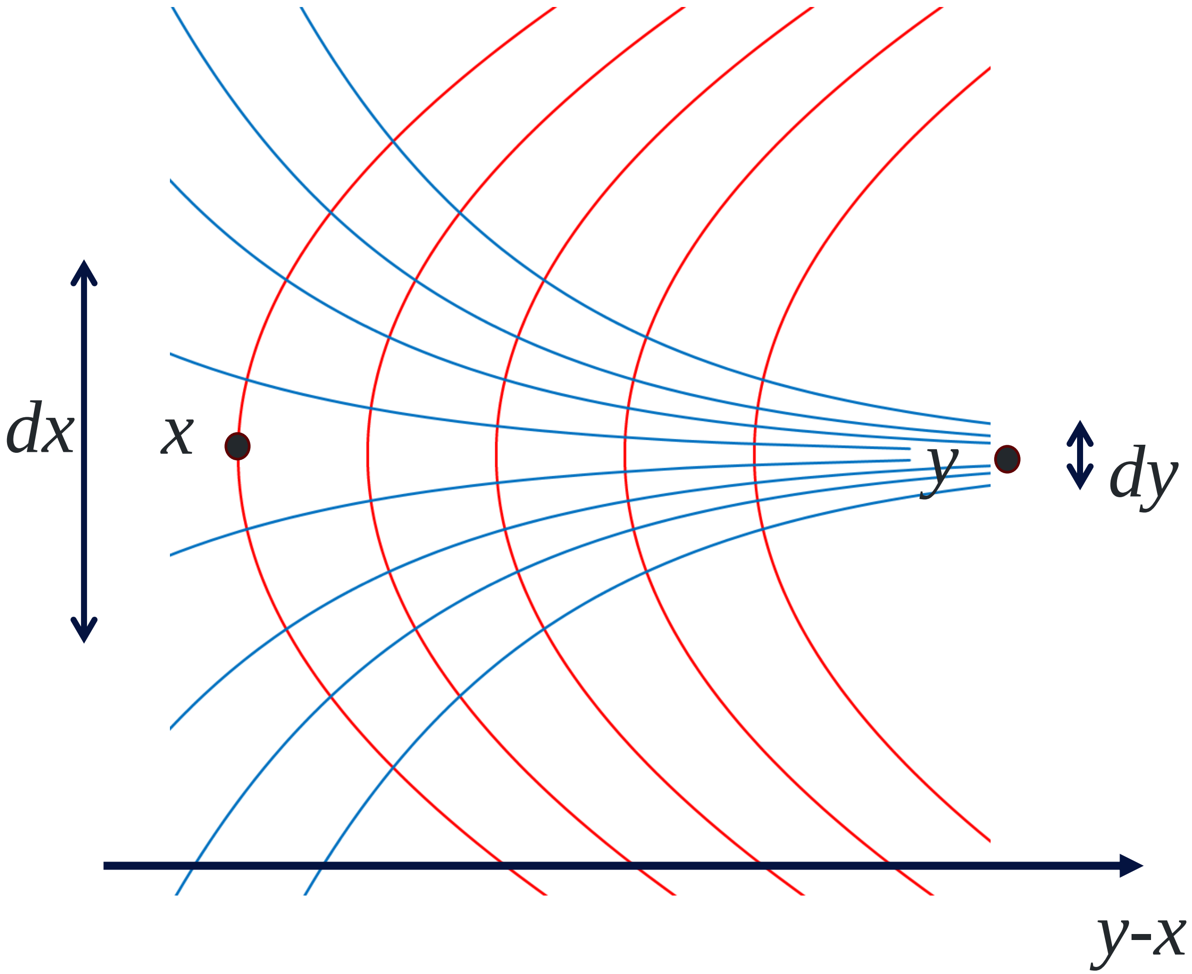
Model Specification and curvature

As data changes, different models are selected from the Model Space.

Geometry and curvature control the sensitivity of the model selection to changing data – this is encoded in the Second Fundamental Form.



The 2nd Fundamental Form - Analysis of Curvature



The key metric here is the classical Second Fundamental Form II of the embedding of model space in data space or in more complex model spaces

$$\frac{\partial \boldsymbol{e}_a}{\partial x^b} = \Gamma^c_{ab}\boldsymbol{e}_c + \mathrm{II}^d_{ab}\boldsymbol{e}'_d$$

This leads to quantification of sensitivity and to the analysis of relative curvature, via Gauss's *Theorema Egregium*

$$||dy|| = ||dx|| \exp\left[\mathrm{II}(dx, dx)(y - x)/||dx||^2\right]$$

$$\Delta R^d_{abc} = \mathrm{II}^d_{pb}\mathrm{II}^p_{ac} - \mathrm{II}^d_{pc}\mathrm{II}^p_{ab}$$

Principles of Model Risk Quantification

- Model Risk has many dimensions each with its own methods - some may not be quantifiable at all - decide what should be quantified and focus on that.
- Model Risk is a relative risk – decide the reference model(s) and the space of models.
- Sensitivity analysis (in its broadest sense) is the basic and universal instrument to quantify model risk.
- Information Divergencies and Relative Entropy are convenient, consistent, mathematically-founded measures, strongly recommended to compare models.
- In practice, model variation can be expressed in financial or operational scales, resulting in quantification of model risk in outcomes familiar and tangible to the bank.

Journal of Risk Model Validation vol.16(3) , Special edition on Model Risk Quantification, Sept 2022.

G.W. Peters, H. Yan and J. Chan; J.L. Breeden and N. Dobrinov; M. Jacobs; D. Arrieta

Conclusions

- Model Risk has many dimensions or aspects and many of these can be quantified by analogy to other kinds of risk.
- The sophistication and approach to Model Risk quantification depends strongly on the analogy used, and on the way models are scoped, described and compared mathematically.
- General principles and pointers to Model Risk quantification are emerging and stabilising in practical applications and in theory, but many opportunities for further development.
- Explore suitable risk measures and model acceptance sets.
  - Are your compliance conditions and model acceptance criteria convex and continuous?
- Explore new kinds of tests and metrics of model development sensitivity
  - Challenge the continuity and iterative stability $M^2=M$ of the modelling process;
  - Challenge model robustness by the analysis of curvature of the model space and the 2nd Fundamental Form.