Forward-Looking Loan Loss Provisioning Under Imperfect Forecasts

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Abstract

I find that banks increasingly rely on macroeconomic forecasts in provisions post implementation of the current expected credit losses (CECL) model, but most bank macroeconomic expectations are not rational. I build a model to study the impact of Kahneman and Tversky's (1972) representativeness heuristic on loan loss provisions, bank lending, and economic output in the context of CECL. In contrast to rational expectations, the model demonstrates that the representativeness heuristic results in overreaction to news, leading to excessive lending and risk-taking in response to good news and overly precautionary lending amid negative macroeconomic shocks. A constant minimum capital adequacy requirement can constrain the effect of excessive optimism, but fails to limit excessive pessimism. The optimal capital requirement must react to the underlying default risk in the economy and also undo the effects of the biases in banks' expectations. Switching to a forward-looking regime in provisions, as well as the potential for overreaction to news, both imply an increased usefulness of a time-varying capital constraint.

1 Introduction

The Great Recession of 2007 exposed the tendency of banks to amplify shocks from the real economy rather than absorb them. In order to counteract this issue, regulatory and accounting adjustments were imposed on banks. In the previous accounting framework, banks recorded provisions using the incurred loss model (ILM). This approach involved recognizing provisions based on past due information, when there was clear evidence that losses were likely to occur, thus surpassing a recognition threshold. However, the effectiveness of the incurred loss framework was criticized for provisions being 'too little, too late', hence contributing to the procyclicality of the banking sector. As a response, the Financial Accounting Standards Board (FASB) introduced the Current Expected Credit Loss (CECL) model.[1] In this new paradigm, banks are required to account for provisions not solely based on past due and current information, but also based on reasonable and supportable forecasts of future economic conditions. This represents a significant departure from the prior retrospective approach, as macroeconomic projections now play a central role in the provisioning framework.

In this paper, I study the impact of tying provisions to future economic conditions under CECL. I start by presenting a few stylized facts about macroeconomic forecasts, which I then use as stepping stones for a theory model, in order to examine how the properties of macroeconomic forecasts affect bank provisions, lending, economic output, and welfare under the new standard, as well as what the optimal capital constraint should be in the context of CECL.

First, I document a noticeable increase in banks' discussions of macroeconomic expectations regarding credit loss estimation methodology and factors influencing loan loss allowances in their financial statements after implementing CECL.

Second, I explore the properties of banks' macroeconomic expectations. An implicit assumption in the regulators' argument, as well as the academic literature, is that economic expectations are rational. For example, Beatty and Liao (2011), argue that banks with timely loss recognition better prepare for future economic downturn defaults. However, banks ability of predicting future macroeconomic conditions is not explicitly tested, and hence the impact of macroeconomic forecasting on bank provisions and losses is not well understood.

Recent accounting papers point to the difficulty in estimating future losses (e.g. Harris et al., 2018, Lu and Nikolaev, 2022). However, as credit losses depend on the state of the

[1] Accounting Standards Update (ASU) 2016-13 (ASC 326)

economy, a more fundamental issues is the difficulty in forecasting future macroeconomic conditions. Despite the predominant paradigm that expectations are rational, abundance of empirical macroeconomic and behavioral finance literature strongly rejects the hypothesis that macroeconomic forecasts are rational. The evidence against rational expectations is consistent across a large range of forecasting agents, across a variety of macroeconomic variables, and across different lengths of forecast horizons. An influential paper within this stream of research is Coibion and Gorodnichenko (2015), who present a novel empirical framework to test the full-information rational expectations hypothesis. The authors apply this approach to U.S. and international data on inflation forecasts by professional forecasters and document systematic deviations of the forecasts from the FIRE benchmark. They document similar results for other economic agents, such as commercial banks, non-financial businesses, and consumers. Building on their methodology, Bordalo et al. (2020) find an overreaction to news of individual forecasts compared to what FIRE would imply. The documented overreaction on an individual level is very interesting because it is at odds with most models previously used to explain flaws in consensus macro forecasts, such as those models related to signal extraction process (Lucas, 1972; Kydland and Prescott, 1982; Woodford, 2003), infrequent information updating (Mankiw and Reis, 2002), and rational inattention (Sims, 2003). Moreover, even in an experimental setting, controlling for incentives, forecasters' information sets as well as the data generating process, it has been shown that forecasters display significant overreaction to recent observations (Afrouzi et al., 2023).

Using the Blue Chip Financial Forecasts survey, I test the properties of macroeconomic forecasts that are provided by banks and professional forecasters. Following the methodology of Coibion and Goridnichenko (2015) and Bordalo et al. (2020), I document that the majority of forecasters exhibit overreaction to news. These results indicate that assuming bank forecasts are rational ignores this widespread overreaction to news. Therefore, in the rest of the paper I take the possibility for biases in expectations seriously.

I follow Bordalo et al. (2020), who reconcile the finding of overreaction to news by Kahneman and Tversky's (1972) representativeness heuristic, formalized by Gennaioli and Shleifer (2010) and Bordalo et al. (2018). The representativeness heuristic implies that agents overweight the future states whose likelihood has increased when news arrive. The source of the deviation from rational updating is that the human mind does not retrieve all possible states, but rather focuses on the more likely ones when forming expectations. Bordalo et al. (2020) find empirical support for the importance of the representativeness heuristic in professional macroeconomic forecasting.

I develop a model of bank lending, in which a representative bank is subject to a capital adequacy constraint and provisions are accounted based on CECL. To capture the empirical finding that bank macroeconomic forecasts exhibit overreaction, I consider macroeconomic expectations which are subject to Kahneman and Tversky's (1972) representativeness heuristic. As benchmarks, I use rational expectations, as well as expectations based on extrapolating from history.

There are two dates of interest: t and $t+1$. At time t, the bank starts with an exogenously fixed amount of equity, raises deposits, and originates a risk-free and a risky loan. The return on the risky loan depends on the future macroeconomic state. The bank observes the current macroeconomic state, forms expectations about the future, and chooses the interest rates for the two types of loans. In line with CECL, the bank accounts for loan loss provisions based on its expected credit losses, and provisions affect its regulatory capital. The bank's decision at time t determines the total amount of lending, aggregate output, and expected surplus in the economy. It also determines the bank's exposure to macroeconomic risk, and hence the probability of bank failure. At time $t+1$, the bank observes if the risky borrower repays or defaults on the loan and accounts for the corresponding profit or loss. At that time, the bank also realizes the return from the safe loan and repays its deposits if it has the funds to do so.

A key feature of the model is the sensitivity of the performance of the risky loan to the state of the economy. As a result, macroeconomic expectations play a key role in the bank's lending decision. In reality, bank borrowers' probabilities of default also depend on the borrower's fixed and time-varying characteristics, which are not necessarily related to macroeconomic factors. Here, I abstract from these details in order to isolate the effect of changes in expected credit losses driven by changes in the macroeconomic conditions. I do this to study the effect of CECL mandating banks to consider future economic conditions in the estimates of expected credit losses, and relate the analysis to the ample empirical evidence that macro forecasts deviate from the full information rational expectations framework.

I obtain the following results. The first-best benchmark in the economy is when the bank possesses enough equity to not be constrained by the minimum capital adequacy requirement and forms expectations rationally. Under rational expectations, optimal bank lending and economic output co-vary with the macroeconomic state. This stems from the fact that the macroeconomic conditions are persistent, hence the current macroeconomic state is informative about the expected future returns. The expected surplus in the economy is maximized and the bank has enough equity to absorb potential loan losses so that there is no risk of bank failure.

Under the representativeness heuristic, good news leads to excessive optimism, which drives more extensive loan origination and larger output in the economy. Nevertheless, because the bank underestimates the default risk when using the representativeness heuristic, risk exposure of the bank is higher than rational, which raises the peril of bank failure. In contrast, overreaction to bad news leads to excessive precautionary behavior in downturns, which may lead to an amplification of the bad shock in the economy by excessively shrinking production and can cause a significant loss of welfare compared to the rational benchmark. These results suggest that reliance on forecasts, when forecasts are subject to biases, can lead to undesirable outcomes. In other words, requiring banks to be forward-looking can cause procyclicality when their forecasts are subject to systematic overreaction to news.

I also compare the results under expectations based on the representativeness heuristic and expectations based on simple extrapolation from current macroeconomic data. The second method is consistent with assuming that the ratio between losses and total loan amounts will remain unchanged in the future. This method was commonly used in practice, but it is subject to criticism expressed in relation to the incurred loss model, according to which banks relied too much on current and past information and did not pay enough attention to expectations about the future. My analysis shows that relying on macroeconomic forecasts, compared to relying on the simple extrapolation, can exacerbate the procyclicality of the banking sector when forecasts are subject to strong overreaction or when the economy is very persistent. This points to the fact that it is not enough for banks to be forward-looking, but that they need to be able to rationally predict the future macroeconomic conditions.

Turning attention to the interaction between the forecasting biases and the requirement for maintaining capital adequacy ratio above a fixed regulatory minimum, not surprisingly, the presence of a capital requirement does not have any impact on the bank's lending decision when its equity is so high that the capital constraint is slack. In such case, the effect of overreaction to economic news is not impacted. When the bank is constrained by the capital adequacy minimum, the requirement tends to limit loan origination under good news, and in that sense, it limits the effect of the representativeness heuristic. Under bad news, loan origination falls and the capital adequacy ratio increases, making the capital requirement slack. Strong pessimism in bad times stemming from the representativeness heuristic, limits the bank's exposure to risk and reduces the probability of bank failure. However, being slack, the capital requirement has no bite in restricting the procyclical reaction of bank lending in the case of downturns, which is too strong compared to the rational reaction. The welfare cost stemming from the forecasting bias can be substantial. This finding supports the concerns expressed by banks in the debate around CECL that the forward-looking regime can exacerbate procyclicality, especially during recessions.

Furthermore, I find that when the bank is constrained by the capital requirement, the timelier provisioning of credit losses under CECL/IFRS 9 compared to the incurred loss framework makes the bank more conservative in the sense that it limits the exposure to the risky borrower. This can limit the amount of excessive lending under good news, thus containing the effect of the representativeness heuristic bias. However, it can also exacerbate the problem of excessive pessimism when bad macroeconomic news arrives. This result echoes the argument, expressed by some banks and politicians, that CECL can increase the cost and lower the availability of credit.

Next, I turn attention to bank regulation. In particular, I study what the bank capital constraint should be under different expectation formation processes. I find that the minimum capital adequacy should be more stringent when the default risk in the economy is higher. However, when banks' expectations are not formed rationally, the capital requirement should also take into account the belief banks hold about the default rate. Holding the rational underlying default risk constant, the optimal capital requirement is decreasing in the perceived risk by banks. In a sense, the regulator should try to correct for the bias in bank forecast through the capital requirement. This finding highlights the interlinkages between the properties of provision numbers and the optimal capital requirement, or between accounting standards and optimal banking regulation, more generally. I also study the implications of the different expectation formation processes on the variation of the capital constraint over time. This is important because CECL/IFRS 9 require provisions to reflect changes in economic conditions on a timely manner. I find that moving from the extrapolation method in provisions to forward-looking provisioning based on rational expectations implies a higher variation in the capital constraint. In other words, moving from the extrapolation method in provisioning to rational expectations, highlights the stronger need for time-varying capital constraint. This is further exacerbated if expectations are forward-looking and subject to a strong representativeness bias. Therefore, switching to a more forward-looking regime in provisions, and the potential overreaction to news, both highlight the time-variation of the regulatory problem, and point to the potential usefulness of time-varying capital-constraint.

This paper is related to several strands of banking regulation, accounting, and behavior macroeconomic literature. It provides a theoretical framework of the link between macroeconomic forecasts, bank provisioning, and lending in CECL, expanding the still scant theoretical literature on this topic (Bouvatier and Lepetit, 2012; Mahieux et al., 2020; Bertomeu et al., 2020; Huber, 2021). A novel feature of this model is explicitly modeling the properties of the macroeconomic forecasts and linking the bank lending decision to economic activity and welfare. To the best of my knowledge, this is the first paper to study how the properties of the forecasts affect the provision numbers, which might be relevant to users of accounting numbers, such as investors and regulators.

This paper also contributes to the growing empirical literature studying the potential procyclical effects of CECL/IFSR 9 (Beatty and Liao, 2011; Abad and Suarez, 2018; Krüger et al., 2018; Covas and Nelson, 2018; Cohen and Edwards, 2017; Chae et al., 2018; Kim et al., 2022, Chen et al., 2022) by highlighting behavior biases in macroeconomic forecasts as a potential source of procyclicality. This paper also supports the concern that future credit losses are difficult to predict accurately (Harris et al., 2018; Lu and Nikolaev, 2022) by drawing attention to the systematic errors in macroeconomic expectations. Although deviations from rationality have gained attention in macroeconomic and finance research, this is less common in the accounting literature—some exceptions are Chan et al. (2004), Koch and Wüstemann (2009), and Kochetova and Salterio (2003) who provide a review of the older literature. Behavior studies in accounting are also mostly limited to the study of judgement and decision-making by managers and auditors, although heuristics are also likely to apply to many other settings. By combining the use of macroeconomic signals and behavior biases in expectations formation, I present novel theoretical predictions about the effects of CECL/IFRS9.

This paper is also related to the finance literature by linking macroeconomic expectations, lending behavior, and credit cycles (e.g. Ma et al., 2021; Bordalo et al. (2018)). However, I look at the role of accounting standards and capital requirements in establishing the mechanism for this relationship rather than being agnostic about the mechanism or linking lending dynamics to fluctuations in asset prices. Lastly, it highlights the interlinkages between the accounting standards and bank regulation in the context of CECL, and suggests an increased need for a time-varying capital constraint post CECL-adoption.

2 Macroeconomic expectations

2.1 Use of expectations

Forecasts made by various economic actors have been demonstrated to be a meaningful indicator of the beliefs that underlie their actions. For example, Greenwood and Shleifer (2014) study the role of investor expectations about aggregate stock returns. They combine data on expectations on aggregate stock returns, both quantitative and qualitative, from six different sources, with diverse surveyed populations (from individual investors, consumers, financial market newsletters and CFOs of large U.S. companies) and different survey questions. The research reveals that expectations of aggregate market returns are highly consistent across the surveys and are correlated with investor fund flows into equity mutual funds, suggesting that the surveyed forecasts are associated with actual investment behavior. Gennaioli, Ma and Shleifer (2015) show that forecasts by corporate chief financial officers about firm earning growth are linked to the firms' investment plans and actual investment. In a survey over the period 1989–2015 on around 1000 firms per year in Japan, Tanaka et al. (2019) document that firms' GDP forecasts are positively and significantly associated with their subsequent input choices, such as investment and employment, as well as sales growth. Forecast accuracy is found to be tightly related to profitability and productivity.

In the banking setting that I study in this paper, the use forward-looking information is stipulated by the accounting standard. Under CECL, banks should recognize all loan losses at their expected value, given reasonable and supportable forecasts of future economic conditions. Macroeconomic conditions are an important factor that affects loan performance, and therefore macroeconomic forecasts are a key ingredient of the estimation of expected credit losses under the new accounting standard.

Kim et al. (2023) provide evidence that banks rely more heavily on forward-looking information following the adoption of CECL, as intended by the new standard. However, their analysis potentially captures both macroeconomic and other forward-looking information. As I am primarily interested in the reliance on macroeconomic forecasts, I conduct textual analysis of bank financial statements, which confirms the increased focus by banks on macroeconomic forecasts after the adoption of CECL. I use filings of 10Ks and 10Qs of banking corporations from the SEC Edgar database, which I match to bank Call Reports from the Federal Deposit Insurance Corporation. I search for references of the annual and quarterly reports to macroeconomic forecasts[2]. Figure 1 plots the references to macroeconomic expectations for CECL-adopting banks and non-adopting banks over time. The group of CECL-adopters only contains banks that adopted the new provisioning method in Q1 2020, while the other group consists of banks that use the incurred loss framework during the whole reported period.[3] We see a sharp increase in the share of banks that discuss macroeconomic expectations in their financial statements post-implementation of CECL. Banks typically include a section on allowances for credit losses, which explains the key drivers of their model of expected credit losses, as well as the key macroeconomic variables upon which it depends. This section often presents the forecast of these variables, in qualitative or quantitative terms, as well as how changes in the macroeconomic forecasts affect the allowance for credit losses. Appendix A contains examples of these sections taken from the annual reports. This evidence suggests that banks have responded to CECL by using forward-looking information to determine provisions. Specifically, I find evidence that banks rely on macroeconomic forecasts to estimate expected credit losses post-implementation of CECL, indicating the importance of studying how macroeconomic forecasts affect banks' provision numbers.

2.2 Properties of macroeconomic expectations

I now explore the properties of macroeconomic forecasts, using data from Blue Chip Financial Forecasts. Blue Chip is an organization which surveys the expectations of panelists, most of which are professional forecasters, financial institutions, insurance companies and wealth management companies. The survey is conducted monthly and the panelists are asked to provide forecasts of real GDP growth, inflation and interest rates for the current quarter and all five future quarters. The sample contains 230 panelists, 99 of which are financial institutions, and runs over the period Jan 1983 - Feb 2023. Although the survey collects institutional forecasts, I use the terms forecasting institution and forecaster interchangeably in this text.

There is evidence that the forecasters in this sample act in line with their disclosed beliefs.

[2]Specifically, I search for forward-looking expressions, related to future macroeconomic conditions, such as mentions of "macroeconomic forecast", "macroeconomic outlook", "macroeconomic scenarios".

[3]ASU 2016–13 was initially set to take effect in January 2020 for all SEC filers, except for smaller reporting companies. Due to the COVID-19 pandemic, the CARES Act provided firms with an option to delay CECL adoption until the earlier of the first date of an eligible financial institution's fiscal year that begins after the date when the COVID-19 national emergency is terminated or January 1, 2022.

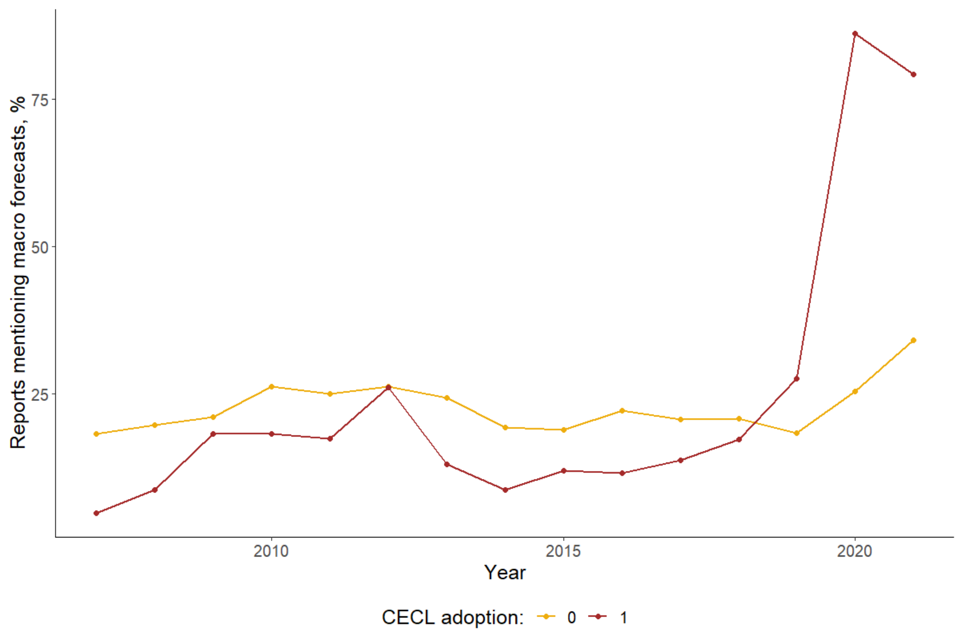


Figure 1: Reference to macroeconomic forecasts in filings of annual and quarterly financial statements

Wang (2021) matches a subset of banks from the Blue Chip data to their balance sheet information from the Call Reports and finds that banks' allocations to Treasuries vary positively and significantly with their subjective expectations of bond returns at the corresponding maturities. Ma et al. (2022) document that bank GDP growth forecasts from the Blue Chip financial forecasts are in line with the baseline projections in the Federal Reserve's FR Y-14A form, which are used by banks for their capital assessments and stress testing. This accumulated evidence indicates that banks rely on the Blue Chip macroeconomic forecasts, which underscores the importance of analyzing the characteristics of these forecasts to understand their implications for banks' decision-making, as well as their role in assessing credit risks and accounting for provisions, in particular.

I test the hypothesis of rational expectations following the methodology of Coibion and Goridnichenko (2015), which studies the relationship between forecast revisions and forecast errors. If the rational expectations hypothesis holds, forecast errors should not be predictable by the information available at the time of making the forecast. Under the hypothesis of full information rational expectations, the forecaster should be taking into account all the available information at the time of forecasting, processing it optimally using Bayesian updating. Therefore, future forecast error should not be systematically related to prior forecast revisions.

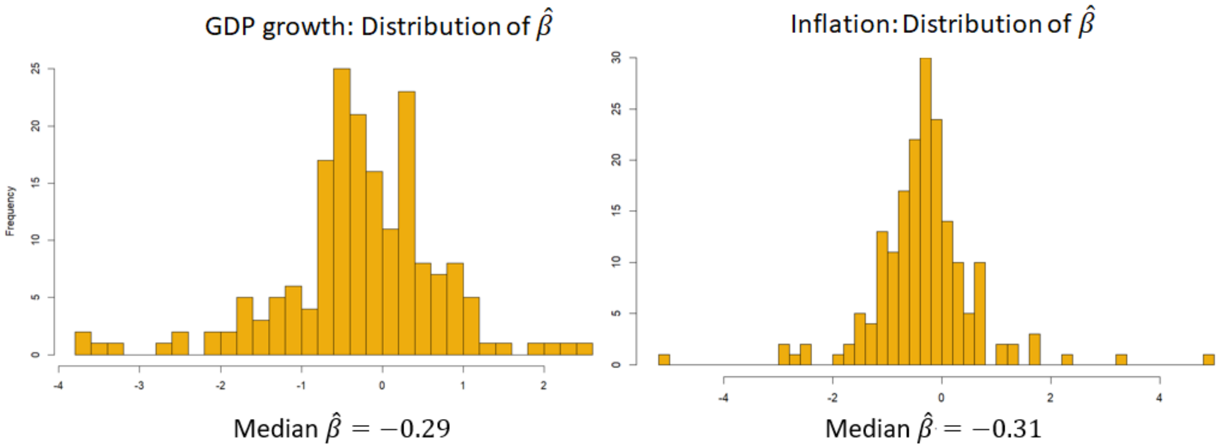


Figure 2: Estimated coefficient: forecast error on forecast revision

Empirically, the predictability of forecast errors can be assessed using the following regression:

$$x_{t+h} - F_t x_{t+h,i} = \alpha^i + \beta^i (F_t x_{t+h,i} - F_{t-1} x_{t+h,i}) + \delta_h + \delta_{var} + e_{t,h,i},\ i = 1, 2, \dots I \qquad (1)$$

where $i = 1, \dots, I$ is an index of the forecaster, $h \in [1, 5]$ is the forecast horizon in quarters, δ_h captures fixed effects for the forecasting horizon. Let $x_{t+h,i}$ denote the realized value of the variable at time $t + h$, while $F_t x_{t+h,i}$ is the forecast of the variable h quarters ahead that is produced by forecaster i at time t, and $F_{t-1} x_{t+h,i}$ is the first lag of this variable, i.e. forecast produced at time $t - 1$. In essence, the equation regresses the forecast error, $x_{t+h} - F_t x_{t+h,i}$, on the magnitude of the revision of the forecast, $F_t x_{t+h,i} - F_{t-1} x_{t+h,i}$. The main coefficient of interest in the regression is β^i. The hypothesis of rational expectations is consistent with $\beta^i = 0$. If $\beta^i \neq 0$, the full information rational expectations hypothesis can be rejected. In particular, finding $\beta^i < 0$ is consistent with overreaction to news, while $\beta^i > 0$ is consistent with underreaction. Rational expectations imply that $\beta^i = 0$ for all i. We can take the median coefficient as indicative of whether the majority of forecasters over- or underreact to news.

Following Bordalo at al.(2020), I estimate Equation (1) separately for each forecaster $i = 1, 2, \dots I$. I do not pool the data for all forecasters into a single regression, as this would impose the same coefficient of reaction to news β^i, which might not be a reasonable assumption in case of heterogeneity in the forecasting properties across forecasters. The two histograms in Figure 2 summarize the estimated coefficients β^i across forecasters. The first panel is based on the forecasts of real GDP growth (SAAR, annualized q-o-q rate), while the second panel is based on the forecasts of inflation (SAAR, annualized q-o-q rate). Both panels show that there is a large mass of the distribution for which β^i differs from zero, and the deviations are economically significant. Judging from the median coefficients, -0.29 for real GDP growth and -0.31 for inflation, we can conclude that the majority of forecasters overreact to news. The histograms offer valuable insight into forecast properties, revealing substantial heterogeneity. More importantly, they serve as evidence that the data does not support the assumption of rational expectations.

After examining each forecaster's behavior individually, the analysis has revealed significant heterogeneity in forecasting behavior. To further explore and identify the most noteworthy differences, I aim to group forecasting institutions that exhibit similar responses to news. Through this categorization, I hope to uncover the primary emerging forecasting patterns. However, it is not clear ex ante how many distinct groups of forecasting behavior exist within the data, and how to group the institutions accordingly. Thus, I utilize a data-driven approach, specifically latent class analysis (LCA), which groups firms into clusters with homogeneous characteristics based on the sign and significance of the association between forecast revisions and forecast errors. The LCA method is especially beneficial since it accounts for the fact that the estimated coefficient connecting forecast revisions and forecast errors may vary in magnitude, significance, and even sign across different subsets of firms.

The LCA model assumes that the data can be characterized by:

$$x_{t+h} - F_t x_{t+h,i} = \alpha + \beta^c (F_t x_{t+h,i} - F_{t-1} x_{t+h,i}) + \delta_i + \delta_h + \delta_v + e_{t,h,i},\ c = 1, 2, \dots C \qquad (2)$$

where δ_i, δ_h and δ_v capture forecasting institution, forecasting horizon and macroeconomic variable of interest (real GDP growth or inflation) fixed effects, $c = 1, 2, \dots, C$ indexes the number of clusters and the the coefficient on forecast revisions β^c varies across clusters.[4] I choose the number of clusters based on the Bayesian Information Criterion (BIC) (Nylund et al., 2007).[5]

Following Larcker et al. (2019), I estimate Equation (2) in two steps. The benefit of this

[4]The LCA model assumes that the dependent variable (here, the forecast error $x_{t+h} - F_t x_{t+h,i}$), is distributed as a finite mixture of normal distributions. The estimation procedure maximizes the likelihood function:

$$L = \prod_{i=1}^{I} \prod_{t=1}^{T} \prod_{h=1}^{5} \left[\sum_{c=1}^{C} \lambda_c \frac{1}{\sqrt{2\pi\sigma_c^2}} \exp\left(-\frac{(x_{t+h} - F_t x_{t+h,i} - (\alpha + \beta^c (F_t x_{t+h,i} - F_{t-1} x_{t+h,i})))^2}{2\sigma_c^2} \right) \right]$$

where where λ_c is the unknown proportion of the sample that is contained in cluster c, σ_c is the standard deviation of the error term withing the cluster, and β^c is the coefficient representing reaction to news within the cluster c.

[5]I increase the number of clusters until there is no further sizable benefit in term of lowering BIC.

two-stage procedure is making sure that only differences in the reaction to news rather than potential differences in the fixed effects determine the clusters, which is my primary interest. In the first step, I partial out the fixed effects for the macroeconomic variables, forecast horizons and forecasting institution (denoted by δ_v, δ_h and δ_i). Second, I run the LCA analysis on the estimated residuals.

In the first stage, I estimate the following two regressions for the forecast error (FE) and forecast revision (FR) as dependent variables:

$$x_{t+h} - F_t x_{t+h,i} = \alpha^{FE} + \delta_v^{FE} + \delta_h^{FE} + \delta_i^{FE} + e_{t,h,i}^{FE}$$
$$F_t x_{t+h,i} - F_{t-1} x_{t+h,i} = \alpha^{FR} + \delta_v^{FR} + \delta_h^{FR} + \delta_i^{FR} + e_{t,h,i}^{FR}$$

In the second stage, I estimate the following regression, using the LCA methodology described above:

$$\hat{e}_{t,h,i}^{FE} = \alpha + \beta^c \hat{e}_{t,h,i}^{FR} + \varepsilon_{t,h,i} \tag{3}$$

The estimated coefficient β^c in the second-stage regression is equivalent to the coefficient of interest β^c in Equation (2). Moreover, I restrain the LCA optimization problem so that all observations pertaining to the same banks are a part of the same cluster. The reason for using this constraint is that the forecasting properties are found to be persistent over time, and incorporating this information leads to a more efficient estimation procedure.[6]

Panel A of Table 1 reports the results from the LCA analysis based on Equation (3), while Panel B shows the size of each cluster. I find evidence that the whole sample of forecasters (columns 1 and 2) consists of a mixture of two clusters, both of which exhibit significant over-reaction to news. This is evident by the negative and statistically significant coefficients on the forecast revision term in both clusters: -0.11 and -0.15 respectively. Focusing attention to banks only (columns 3 and 4), we see that the forecasting properties are slightly closer to rational. The sample of banks is partitioned into two clusters, revealing that approximately 27% of banks comprise the first cluster where forecast errors are not predictable by forecast revisions. This is evident by the statistically insignificant estimated coefficient $\hat{\beta}^c = -0.03$, which aligns with

[6]As a robustness check, I compare the LCA classification based on the whole sample (up to February 2023) with that based on the samples up to 2019, and up to 2009. I find that that 97.5% and 80% respectively of the forecasters are classified in the same clusters based on the data up to 2019, or using the data up to 2009, supporting the argument that forecasting behavior is persistent.

Table 1: Latent Class Analysis of the Predictability of Forecast Errors

	(1)	(2)	(3)	(4)
Panel A. LCA estimation results				
	Dependent variable: Forecast error			
	Full sample		Banks	
	Cluster 1	Cluster 2	Cluster 1	Cluster 2
Forecast revision	-0.1123**	-0.1507***	-0.0278	-0.1035**
	(0.0372)	(0.0251)	(0.0514)	(0.0383)
Intercept	0.0000	0.0000	0.0000	0.0000
	(0.0351)	(0.0129)	(0.0519)	(0.0192)
Panel B. Cluster size				
Number of forecasters	60	146	23	62
% of total forecasters	29%	71%	27%	73%

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

rational expectations for this group of banks. The remaining 73% of banks, however, show overreaction, as indicative by the negative and statistically significant coefficient on forecast revision ($\hat{\beta}^c = -0.10$).[7] Overall, for the whole sample, as well as for the sample of banks, we see that the predominant expectation formation process is not rational, but rather subject to overreaction to news.

The analysis in this section revealed considerable heterogeneity in forecast properties and highlights the prevalence of overreaction to news. Assuming bank forecasts are rational ignores the widespread deviations from such behaviour in the data. Therefore, in the rest of the paper I take the possibility for biases in expectations seriously, and study the implications for bank provisions, bank lending, and aggregate economic outcomes.

3 The model

I present a model of bank lending and loan loss provisioning, in which expectation formation is subject to deviations from Bayesian updating. There are two sectors in the model: an entrepreneurial and a banking sector. The entrepreneurial sector consists of two representative

[7] Table 6 in the Appendix shows key descriptive statistics for the two clusters of banks, based on their 10K filings on Compustat for the fiscal years ended after March 31, 2022. Although the sample is small to draw definitive conclusions (19 banks), the summary statistics indicate that the banks with expectations close to rational tend to be bigger, more profitable and more likely to be incorporated in the US compared to the group that exhibits overreaction to news.

agents who seek loans to fund their projects: one has a risk-free project, and one has a risky project. The banking sector consists of a representative bank that decides whether or not to provide loans to the entrepreneurs and set the respective interest rates. The bank is subject to a capital requirement, which resembles the Basel Accord. In this section, my focus is not on determining the optimal capital requirement from a regulatory standpoint. Instead, I consider it to be a predetermined factor. At this stage, I aim to reflect the present regulatory landscape in the United States, where the capital requirement remains stable and unchanging over time. However, in Section 6, I examine the optimal capital constraint, taking into account the impact of expectations on provisions.

Success of the risky project depends on the macroeconomic state. The macroeconomic state at date t, denoted by x_t, follows an AR(1) process:

$$x_t = \rho x_{t-1} + u_t$$

where $x_0 = 0$, $\rho \in (0, 1)$ is a known constant and the error terms u_t are iid, $u_t \sim N(0, \sigma_u^2)$. Throughout the paper, I refer to the error terms in this process as macroeconomic shocks and macroeconomic innovations interchangeably. Notice that the coefficient, ρ, governs the persistence of the macroeconomic shocks over time.

3.1 Expectation formation

As shown more precisely in the following sections, a key ingredient in the bank's lending decision are its expectations of the future macroeconomic state $\mathbb{E}[x_{t+1}]$. It is directly linked to the probability of default of the risky project, hence with the bank's expected profits and provisions. Therefore, in this section I show how expectations are formed.

Following Bordalo et al. (2020), I consider expectation formation under both rational expectations and the representativeness heuristic.

3.1.1 Rational expectations and full information

As a benchmark, I first consider expectation formation under full information and rational expectations. In this case, at each date the bank observes the current and past macroeconomic states, $X_t = \{x_t, x_{t-1}, ..., x_0\}$. Let $f(x_{t+T}|X_t)$ denote the distribution of $x_{t+T}|X_t$. Considering that the evolution of the macro state is governed by the process $x_{t+T} = \rho^T x_t +$

$\sum_{i=0}^{T-1} \rho^i u_{t+T-i}$, for $T \geq 1$, we can see that the distribution of $x_{t+T}|X_t$ is a sum of normally distributed variables and, therefore, also normal. Proposition 1 characterizes this distribution.

Proposition 1. *Under the information set $X_t = \{x_t, x_{t-1}, ..., x_0\}$ and rational expectations, the distribution $f(x_{t+T}|X_t)$ is normal and characterized by:*

$$\mathbb{E}[x_{t+T}|X_t] = \rho^T x_t$$
$$Var(x_{t+T}|X_t) = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2$$

3.1.2 Representativeness heuristic and full information

Kahneman and Tversky have conducted groundbreaking research on decision-making under uncertainty. They have documented numerous flaws of the human mind when it comes to statistical thinking and evaluating probabilities (Kahneman and Tversky, 1972, 1974, 1983). In this section, I allow expectations to rely on Kahneman and Tversky's representativeness heuristic for two main reasons. First, they show that the heuristic is psychologically ingrained in the human mind, has survived substantial experimental scrutiny over time, and is likely to affect how expectations are formed in many settings. Even sophisticated statistical forecasting models tend to depend on exogenous assumptions that are subject to the forecaster, thus opening up room for the effects of heuristics. Second, unlike the models of sticky information (Mankiw and Reis, 2002), noisy information (Lucas, 1972) or rational inattention (Sims, 2003), the representativeness heuristic is able to reconcile recent empirical findings about macroeconomic forecasts: namely, the observed overreaction on individual level and underreaction on aggregate, heterogeneity across macroeconomic variables, and a link between the persistence of the process and magnitude of bias in the forecast (Bordalo et al., 2020).

I am using the formalization of the representativeness heuristic by Gennaioli and Shleifer (2010) and Bordalo et al. (2020). The representativeness heuristic implies that agents overweight the future state whose likelihood has increased the most as a result of news relative to before observing the news. Equivalently, this implies underestimating or even ignoring states that are less likely given the new information. In the setting of this paper, this implies that if the latest macroeconomic news is good, agents will overestimate the possibility for a continued expansion in the future (as good news is more representative of expansions) and underestimate the possibility of a recession, compared to what Bayesian updating would imply. The source of the deviation from rational updating is that the human mind does not retrieve all possible states, but rather focuses on the more representative ones when forming expectations.

To illustrate this heuristic, consider the following example. Suppose there are three possible macroeconomic states in the future period $(t+1)$: (1) recession in which GDP falls by 5%, (2) the GDP stays unchanged, and (3) an economic boom in which GDP grows by 5%. Once the macroeconomic signal at time t is observed, suppose there are three possible scenarios about the information in the signal: (1) No news, i.e. the realized signal fully corresponds to the expectation from the previous period $t-1$, (2) Good news, i.e. the realized signal is better than the expectation from the previous period, and (3) Bad news, i.e. the signal falls short of the expectation. Table 2 presents the conditional distributions of the macroeconomic states under each of these scenarios.

Table 2: Distribution of macroeconomic states, an example

	State		
	Recession	No change	Boom
GDP growth	-5%	0%	5%
	Pr(State|News)		
No news	0.10	0.50	0.40
Good news	0.05	0.45	0.50
Bad news	0.20	0.60	0.20
	Representativeness factor		
No news	1.0	1.0	1.0
Good news	0.5	0.9	1.3
Bad news	2.0	1.2	0.5

Under rational expectations, the forecaster considers all possible states whose probabilities are presented in the table. In the case of no news: $\mathbb{E}[\text{GDP growth}|\text{No news}] = 1.5\%$, in case of good news, $\mathbb{E}[\text{GDP growth}|\text{Good news}] = 2.25\%$ and in case of bad news, $\mathbb{E}[\text{GDP growth}|\text{Bad news}] = 0\%$.

Using the representativeness heuristic, the forecaster considers the so-called "representativeness" of the state once new information has become available, which is defined as the ratio between the likelihood of the state given new information and the likelihood given no new information. The representativeness factor tries to capture to what extent the likelihood of a state has changed due to the latest information. More formally, for any macroeconomic state $x \in \{\text{Recession}, \text{No change}, \text{Boom}\}$ and any scenario of $News \in \{\text{No news}, \text{Good news}, \text{Bad news}\}$, the representativeness factor is defined as:

$$R(x) = \frac{f(x|\text{News})}{f(x|\text{No news})}$$

The forecast using the representativeness heuristic weighs the distribution of the states by their representativeness using the distorted posterior:

$$f^{\theta}(x_t|X_t) = f(x_t|X_t)R(x_t)^{\theta}\frac{1}{Z_t}$$

where $f(x_t|X_t)$ is the underlying Bayesian conditional distribution, $R(x_t)$ is the representativeness factor of each state, and Z_t is a normalization factor ensuring that $f^{\theta}(x_t|X_t)$ integrates to 1. The parameter $\theta \geq 0$ denotes the extent to which the forecast relies on the representativeness heuristic. In particular, when $\theta = 0$, the expectation formation process coincides with the rational one. However, when $\theta > 0$, forecasts that are based on the representativeness heuristic overestimate highly representative states and underestimate unrepresentative states. In the example above, forecasts based on the representativeness heuristic under good news overestimate the possibility for an economic boom compared to the rational forecast because good news makes this state more likely. For example, for $\theta = 0.5$, $E^{\theta}[\text{GDP growth}|\text{Good news}] = 2.6\%$, which exceeds the rational forecast and demonstrates that there is an overreaction to good news. Similarly, under bad news, using the heuristic leads to overweighting the recession and neutral state and underweighting the booming state versus the rational forecast. For example, for $\theta = 0.5$, $E^{\theta}[\text{GDP growth}|\text{Bad news}] = -0.7\%$, which is much lower that the rational forecast and again demonstrates overreaction to news.

Such behavior is documented in numerous studies. Here, the more representative states are overweighted and the less representative ones are underweigthed. Yet, in extreme cases, only a few or just one of the most representative states are retrieved in memory, while the least representative ones are completely ignored. This is also consistent with the empirically documented overreaction to news in macroeconomic forecasts.

Moving beyond the simplified example above, the representativeness heuristic is applied to the model in the following way. The distribution of $x_{t+T}|x_t$, perturbed by the representativeness heuristic is:

$$f^{\theta}(x_{t+T}|X_t) = f(x_{t+T}|X_t)\left[\frac{f(x_{t+T}|X_t)}{f(x_{t+T}|X_t = \rho x_{t-1})}\right]^{\theta}\frac{1}{Z}$$

Proposition 2. *Under the information set $X_t = \{x_t, x_{t-1}, \ldots, x_0\}$, the distribution of $x_{t+T}|X_t$, perturbed by the representativeness heuristic, is normal. Let $E^{\theta}(x_t + T|X_t)$ and $Var^{\theta}(x_{t+T}|X_t)$ denote the mean and variance of this perturbed distribution, and we have the following:*

$$\mathbb{E}^{\theta}(x_{t+T}|X_t) = (1+\theta)\mathbb{E}(x_{t+T}|X_t) - \theta\mathbb{E}(x_{t+T}|X_{t-1}) = \rho^T(x_t + \theta u_t)$$
$$Var^{\theta}(x_{t+T}|X_t) = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2$$

Note that when $\theta = 0$, i.e. when the agent puts no weight on the representativeness factor, this distribution coincides with the one under rational expectations.

3.1.3 Extrapolation

As a benchmark, I also consider simple extrapolation as an expectation formation process:

$$E^{extr}[x_{t+1}|X_t] = x_t$$

This expectations formation process is motivated by banks' actual practice and shares features that were grounds of criticism towards the incurred loss provisioning regime. Some papers and regulators were pointing out that, under incurred loss provisioning, banks used to overweigh the current macroeconomic information by simply extrapolating recent data. It might be that the forward-looking regime, by stipulating banks to consider macroeconomic forecasts, makes them move away from simple extrapolation and rely more on macroeconomic forecasts. In the following sections of the paper, I will explore how such a potential shift affects lending and other related economic outcomes.

3.2 Entrepreneurial sector

There are two types of entrepreneurs, each type differing from each other by their project's sensitivity to the macroeconomic state. Apart from this sensitivity, entrepreneurs are identical.[8]

[8]The assumption that the production function, and hence demand for credit, is identical for both types of entrepreneurs is not essential to the analysis. The main interest in the paper is the provision of loans to the risky entrepreneur, as macroeconomic conditions change. What is key, is that one of the projects is not subject to

Each entrepreneur has no wealth and seeks investment from a bank to finance their projects. At time t, each entrepreneur i seeks funding to finance the setup costs, K_{it}, and invests the raised funding into production. At time $t+1$ the return from the project is realized. The production technology of the entrepreneurs is identical and presented by the following function:

$$Y_{it} = AK_{it}^{\alpha}$$

The term AK^{α}, where $\alpha \in (0, 1)$, represents a diminishing returns to scale technology, which can be interpreted as an investment technology with adjustment costs. The productivity parameter $A > 1$ is a fixed constant, which is large enough so that the net present value of the entrepreneurs' projects are positive.

The project of the first entrepreneur is risk-free, while the one of the second entrepreneur is risky. There is a chance that the second entrepreneur is hit by a shock, in which case they are unable to sell their production at $t+1$, and the entrepreneur is unable to repay the loan.[9] The sensitivity of the risky entrepreneur is indexed by the parameter γ, which is exogenous. This risk parameter shows that if the macroeconomic state falls below this parameter, the entrepreneur is unable to sell its production at time $t+1$. Lower levels of γ indicate lower levels of risk, in the sense that only more adverse macroeconomic states can impede production. Therefore, the probability of default of the second type is $h_t \equiv P(x_{t+1} \leq \gamma | X_t)$. The entrepreneur repays the loan if $x_{t+1} > \gamma$ and defaults if $x_{t+1} \leq \gamma$.

Suppose the entrepreneur is risk neutral. Taking the interest rate offered by the bank as given, the entrepreneur chooses the amount of investment (which is the same as the amount of loan) in order to maximize his or her expected utility:

Entrepreneur 1 (risk-free project): $$\max_{K_{1t}} AK_{1t}^{\alpha} - K_{1t} - r_{1t}K_{1t}$$

Entrepreneur 2 (risky project): $$\max_{K_{2t}} (AK_{2t}^{\alpha} - K_{2t} - r_{2t}K_{2t})(1 - h_t)$$

macroeconomic risk, which is the only risk in the model. The role of the risk-free sector is to allow for the bank to invest in a risk-free asset, and not necessarily collapse in case the risky loan defaults. Moreover, the ratio of risky to risk-free loans will provide a measure of the risk exposure of the bank.

[9] To provide a more concrete example, imagine that at time $t+1$ consumers are hit by a shock, such as an unemployment shock or a shock to their wealth. The shock prevents the consumers from buying the entrepreneur's product, hence his or her production perishes.

Optimal investment for both types of entrepreneurs, $i = 1, 2$, follows the same rule:

$$K_{it} = \left(\frac{1 + r_{it}}{\alpha A}\right)^{-\frac{1}{1-\alpha}}$$

This equation shows a downward sloping demand for loans, which is the same for both types of entrepreneurs regardless of their different exposure to risk. Because of limited liability and no wealth, the entrepreneurs derive utility only in the upside case (i.e. when production is sold and profits are realized), hence their optimal investment decision is not directly affected by the macroeconomic state. As we will see below, the risky entrepreneur is indirectly affected by the macroeconomic conditions, because the bank prices the state-dependent default risk within the interest rate for the loan.

3.3 Banking sector

The banking sector consists of a representative bank. The bank's balance sheet at any date t is the following.

Assets	K_{1t}, K_{2t}
Liabilities	D_t
Equity	E_t

The assets include the amount of the risk-free loan (K_{1t}) and the risky loan (K_{2t}). They are funded by equity (E_t) and deposits (D_t). When the bank accounts for provisions, the loan loss provision lowers the net value of the loan K_t, which translates in a loss that lowers the amount of equity. The bank balance sheet identity $K_{1t} + K_{2t} - LLP_t = D_t + E_t - LLP_t$ holds at any date t.

The timing of the model is the following. There are two dates of interest: t and $t + 1$. At date t, the bank observes the information set X_t, which contains the current macroeconomic state x_t, as well as all history of the previous states back to date 0: $X_t = \{x_t, x_{t-1} \ldots, x_0\}$. The bank does not forget the history, so as new macroeconomic information becomes available over time, the bank's information set expands.

The actions the bank can take are now specified. At t, the bank starts with an exogenously given equity, E_t. The bank cannot issue equity, and its amount is only affected by profits and losses over the two periods of interest, t and $t + 1$. At time t, the bank can originate deposits. For simplicity, I assume that deposits are inelastically supplied, and the interest rate is normalized to zero.[10] At time t, the bank also chooses the interest rates on loans, which can be differentiated across the different types of entrepreneurs: r_{it}. The bank indirectly chooses the amount of loans by setting their interest rates, and the amount of deposits that it raises is a residual term: $D_t = K_{1t} + K_{2t} - E_t$. The bank also accounts for any necessary loan loss provisions.

At time $t+1$, the bank realizes payoff from the risk-free loan at the amount $r_{1t}K_{1t}$. Furthermore, the bank observes whether the risky entrepreneur either repays or defaults on the loan. Formally, at date $t+1$, the loan generates a return of the amount $r_{2t}K_{2t}$ if $x_{t+1} > \gamma$, and a loss of K_{2t} otherwise.

The bank is subject to an exogenous capital adequacy requirement set by the regulator. The capital requirement resembles the Basel Tier 1 capital ratio, which postulates that the ratio of equity to risk-weighted assets should exceed a threshold: $\frac{E_s}{RWA_s} \geq \xi$, where E_s denotes the bank's equity, and RWA_s denotes the amount of the bank's risk-weighted assets at date s. The capital requirement should hold in all time periods. According to the Basel Accord, risk weighted assets are calculated by assigning assets to different risk categories, with weights between 0 and 1 depending on the risk. Here, the risk weight of the risk-free loan is assumed to be 0 while that of the risky loan is 1; hence, the capital requirement can be expressed as:

$$\frac{E_t - LLP_t}{K_{2t} - LLP_t} \geq \xi$$

3.4 The bank's problem under the expected credit loss accounting regime

Given the information at time t, the bank maximizes its expected cumulative profit by choosing the interest rates on loans, making sure the capital requirement is fulfilled.

The bank's problem is:

$$\max_{r_{1t}, r_{2t}} \left(r_{1t}K_{1t} + (1-h_t)r_{2t}K_{2t} - h_tK_{2t}\right)$$

[10] If the deposit interest rate is non-zero, the optimal interest rate on loans increases one-to-one by the deposit interest rate. Therefore, we can think of the current version of the loan interest rates offered by the bank as rates, net of the interest paid on deposits. The deposit interest rate will be priced in the interest paid by the entrepreneurs, limiting the demand for loans, output and entrepreneurial surplus, while some surplus is transferred to the depositors. However, the direction in which lending and total output vary with the macroeconomic state remains unchanged. I have also considered a version of the model in which the supply of loans is elastic, and the investment in the risk-free asset is residual ($K_1 = D + E - K_2$). This version of the model is equivalent, but redistributes some surplus from the bank to the depositors, which are not explicitly studied now.

subject to:

$$\text{Capital adequacy:} \qquad \frac{E_t - LLP_t}{K_{2t} - LLP_t} \geq \xi$$

$$\text{Loan demand:} \qquad K_{it} = \left(\frac{1 + r_{it}}{\alpha A}\right)^{-\frac{1}{1-\alpha}} \qquad i = 1, 2$$

The bank's decision determines the relative exposure to the risky loan. This, combined with the return on the risk-free loan and the level of equity, determines the bank's capacity to absorb losses on the risky loan and the probability of bank failure.

Under the expected credit loss provisioning method, the bank accounts for a loan loss provision at the amount of the expected credit loss. The latter depends on the macroeconomic information at time t through its effect on the expected future macroeconomic state $E[x_{t+1}|X_t]$. The bank accounts for the expected loss as a loan loss provision at time t: $LLP_t = h_t K_{2t}$. Therefore, the capital adequacy requirement is equivalent to:[11]

$$E_t - h_t K_{2t} \geq \xi(1 - h_t)K_{2t}$$

3.4.1 Unconstrained case

If there are no capital requirements, or equity is large enough so that the capital adequacy constraint is not binding, the following system characterizes the solution to the bank problem and its corresponding lending and output levels:

$$r_{1t} = \frac{1}{\alpha} - 1 \qquad\qquad r_{2t} = \frac{1}{\alpha(1-h_t)} - 1$$

$$K_{1t} = \left(\frac{1}{\alpha^2 A}\right)^{\frac{1}{\alpha-1}} \qquad\qquad K_{2t} = \left(\frac{1}{\alpha^2 A}\frac{1}{1-h_t}\right)^{\frac{1}{\alpha-1}}$$

$$Y_{1t} = A\left(\frac{1}{\alpha^2 A}\right)^{\frac{\alpha}{\alpha-1}} \qquad\qquad Y_{2t} = A\left(\frac{1}{\alpha^2 A}\frac{1}{1-h_t}\right)^{\frac{\alpha}{\alpha-1}}$$

We can view this solution to the problem as first-best. The level of bank equity is high

[11] Note that at $t+1$ equity evolves in the following way: $E_{t+1} = E_t - LLP_t + \pi_{t+1}$, where π_{t+1} is the profit of the bank at $t+1$, i.e. bank equity changes only with the amount of profit, which is generated at time $t+1$. Therefore, if $E_t - LLP_t/K_{2t} - LLP_t \geq \xi$, in expectation the capital constraint will also be fulfilled at time $t+1$, i.e. $\mathbb{E}[E_{t+1}/K_{2t+1}|X_t] \geq \xi$, because the bank would set its expected profits above zero. More precisely, if the bank sets high enough interest rates, it effectively rejects all loan applications and originates no loans, $K_{1t} = K_{2t} = 0$, in which case $\mathbb{E}[\pi_{t+1}|X_t] = 0$. Therefore, we can be sure that if the capital requirement is fulfilled at time t, it is expected to be fulfilled at time $t+1$ as well: $\mathbb{E}[E_{t+1}/(K_{2t} - LLP_t)|X_t] = \mathbb{E}[(E_t - LLP_t + \pi_{t+1})/(K_{2t} - LLP_t)|X_t] \geq \mathbb{E}[(E_t - LLP_t)/(K_{2t} - LLP_t)|X_t]$. Therefore, only the capital constraint for time t appears in the optimization problem.

enough so that the bank sets its interest rates optimally, the level of investment generates the highest possible return, and the total expected surplus in the economy is maximized.

The interest rate is inversely related to the default risk of the entrepreneur. As the risk increases, the bank requires a higher interest rate as compensation for the expected loss, which lowers the demand for the loan. As a result, an increase in risk results in a lower amount of loan origination. The interest rate is also inversely related to α, which is related to the demand elasticity for loans. As $\alpha \to 1$, demand is infinitely elastic, which approaches the case in which the banking sector is perfectly competitive and the bank simply breaks even ($(1 - h_{it})r_{it} = h_{it}$). The lower the interest rate, the higher the amount invested by the entrepreneurs, which increases production.

Aggregate output is:

$$Y_t = A\left(\frac{1}{\alpha^2 A}\right)^{\frac{\alpha}{\alpha-1}}\left(1 + \left(\frac{1}{1-h_t}\right)^{\frac{\alpha}{1-\alpha}}\right)$$

Assuming the risky entrepreneur gains 0 utility if their production is not realized on the market at $t + 1$, total expected surplus in the economy is:

$$S_t = (Y_{1t} - K_{1t}) + (h_t Y_{2t} - K_{2t})$$

Share of risky lending in bank's portfolio:

$$R_t = \frac{K_{2t}}{K_{1t}1 + K_{2t}} = \frac{(1-h_t)^{\frac{1}{1-\alpha}}}{1 + (1-h_t)^{\frac{1}{1-\alpha}}}$$

Expected credit losses:

$$ECL_t = h_t K_{2t}$$

I will consider the risk that the bank will fail to repay its deposits at time $t+1$. If the risky entrepreneur repays its loan (i.e. when $x_{t+1} > \gamma$), the bank will be able to repay its deposits. However, in case the risky loan defaults (i.e. when $x_{t+1} < \gamma$), the bank fails if $(1+r_{1t})K_{1t} < D_t$. This happens when the loss on the risky loan cannot be covered by equity and the return on the risky loan, i.e. when $E_t + r_{1t}K_{1t} < K_{2t}$

$$P(bankfailure) = P(x_{t+1} < \gamma)\mathbb{1}\{E_t + r_{1t})K_{1t} < K_{2t}\}$$

If equity is high enough so that $E_t > K_{2t} - r_{1t}K_{1t}$ for all default probabilities h_t, then there is no risk for the bank to fail. However, if equity is not high enough (even if above the minimum capital adequacy ratio), there are parameter values for which the probability of bank failure is non-zero. In particular, when the interest rate on the risk-free loan is low, there exists default probability h, such that $r_{1t} < (1-\xi)(1-h_t)^{\frac{1}{1-\alpha}}$, i.e. $E_t < K_{2t} - r_{1t}K_{1t}$ even if $E_t \geq \xi K_{2t}$. Going forward, I will assume that $(1-\xi) > \frac{1-\alpha}{\alpha}$, so that the probability of bank failure is non-zero.[12]

3.4.2 Constrained case

If the capital adequacy constraint is binding, the solution to the problem is the following:

$$\begin{aligned}
r_{2t} &= \frac{1 + \lambda^{FL}(\xi + (1-\xi)h_t)}{(1-h_t)\alpha} - 1 \\
K_{2t} &= \frac{E_t}{h_t + \xi(1-h_t)} \\
\lambda^{FL} &= \frac{1}{\xi + (1-\xi)h_t}\left(\alpha^2 A(1-h_t)\left(\frac{\xi + (1-\xi)h_t}{E_t}\right)^{1-\alpha} - 1\right)
\end{aligned}$$

where $\lambda^{FL} > 0$ denotes the Lagrange multiplier, which corresponds to the capital requirement. See more details in the Appendix.

Lending is positively related to the amount of equity and negatively related to the capital adequacy threshold. Higher equity allows the bank to originate more loans without violating the capital adequacy requirement. Increasing the capital adequacy threshold (ξ) has the opposite effect. The expectation about the probability of default on the risky loan (h_t) matters, as the bank requires a smaller amount of loan loss provisions, which relaxes the capital adequacy constraint and allows the bank to originate more loans. On the other hand, as the default risk increases, the required provisions also increase. This limits the amount of risky loan origination, but only up to the point when first-best risky lending falls so much that the capital constraint is no longer binding.

Note that the solution to the constrained case is sub-optimal. In particular, the bank sets interest rates above the first-best, hence limiting the total level of lending. Therefore,

[12]For example, $\xi = 0.1$, $\alpha = 0.8$

investment, output and total expected surplus falls short of the first-best.

Proposition 3. *When the bank is constrained by the minimum adequacy ratio:*

$$r_{2t} > r_{2t}^{FB} \qquad K_{2t} < K_{2t}^{FB} \qquad Y_{2t} < Y_{2t}^{FB} \qquad S_{2t} < S_{2t}^{FB}$$

We can think of the lack of sufficient equity in the economy as a friction that exposes the banking sector to the risk of failure, in the sense of it being unable to absorb potential losses from originating risky loans, and being unable to repay its deposits. Minimum capital adequacy mitigates the probability of bank failure, but leads to a deviation from optimal lending behavior.

The problem is constrained when:

$$C^{FL} \equiv \alpha^2 A(1-h_t)\left(\frac{\xi+(1-\xi)h_t}{E_t}\right)^{1-\alpha} - 1 > 0$$

In general, as the default risk rises, the capital constraint becomes more slack.[13] The reason is that when the risky project has a higher default risk, the bank chooses to set higher interest rates, which lowers risky loan origination and results in a higher capital adequacy ratio. As the capital adequacy threshold increases, the more likely the capital adequacy constraint becomes binding, in the sense that a wider range of values of the parameters are consistent with $C^{FL} > 0$. In contrast, when the initial equity (E_t) increases, the constraint is relaxed.

4 The role of macroeconomic expectations in bank lending

In this section, I study how the expectation formation process affects loan origination. In particular, I am interested in how a bank's loan supply is affected by time t macroeconomic innovation u_t, as this term is persistent over time and affects the macroeconomic state in the future. For example, if there is a negative macro shock at time t, the bank can expect the time $t+1$ macro state and loan repayment to be worse compared to the baseline case of no news (i.e. when $u_t = 0$).

4.1 Macroeconomic expectations

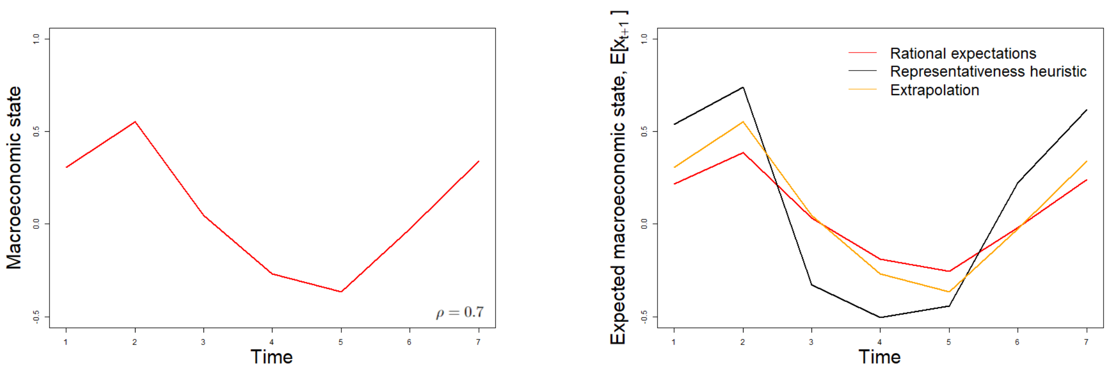
The mean and variance of the distribution of $x_{t+1}|X_t$ under the different expectation formation processes are summarized in Table 3. Figure 3 illustrates how expectations vary with changes

[13] See the Appendix for details.

in the macroeconomic state, based on a simulation of the model. In the simulation, the exact same bank problem is simulated for different points in time s, as if the bank is born with E_s equity and makes lending decisions at time s, realizes payoffs and repays its deposits at time $s+1$, and dies. A new identical bank is born at time $s+1$, and the only difference to the problem is that the macroeconomic state has changed, implying different default probability for the risky entrepreneur.

Table 3: Expectation formation

Expectation formation setting	$\mathbb{E}^{\theta}[x_{t+1}\|X_t]$	$Var^{\theta}(x_{t+1}\|X_t)$
Rational expectations	ρx_t	σ_u^2
Representativeness heuristic	$\rho(x_t + \theta u_t)$	σ_u^2
Extrapolation	x_t	σ_u^2



(a) Current macroeconomic state

(b)Expectations of future macro state, $\mathbb{E}[x_{t+1}|X_t]$

Figure 3: Current macroeconomic state and macroeconomic expectations

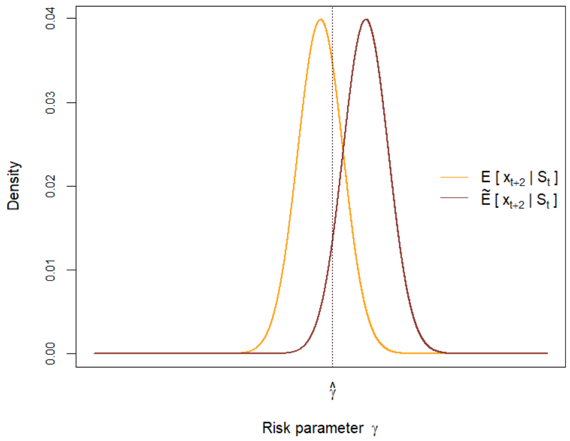
When the bank uses the representativeness heuristic in the expectation formation process, the mean of the distribution can either exceed or fall short of the rational expectations one, depending on the sign of the macroeconomic news u_t. In case of good news, due to overreaction, the distribution of the future state $x_{t+1}|X_t$ will shift to the right, leading to an underestimation of the default probability of the risky entrepreneur. This will result in a stronger loan origination compared to what the rational expectations imply.

4.2 The link between macroeconomic expectations and entrepreneurial probability of default

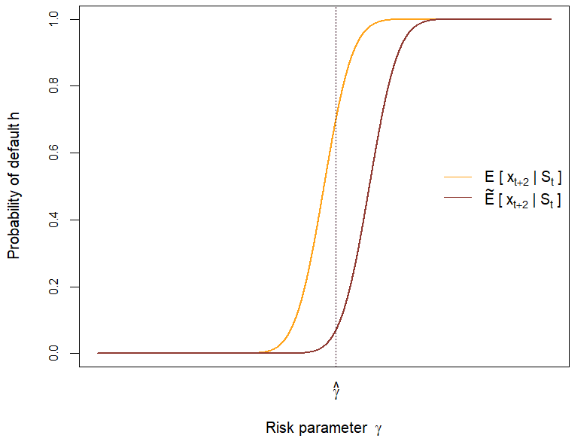
As discussed above, lending positively depends on the loan repayment expectation, both through determining the optimal level of lending under a slack capital constraint and through affecting when the constraint is binding. Furthermore, it is important under the forward-looking accounting regime even when the capital adequacy constraint is binding. Therefore, let us now see how the expectation formation process affects the repayment probability and, consecutively, lending.

Proposition 4. *For the probability of default, defined as* $h_t \equiv \mathbb{E}^\theta[\mathbb{1}\{x_{t+1} \leq \gamma\}|X_t] = \int_\infty^\gamma f^\theta_{\tilde{x}_{t+1}|X_t}(x_{t+1})dx_{t+1}$*, the following holds:*

$$\frac{\partial h_t}{\partial \mathbb{E}^\theta[x_{t+1}|X_t]} \leq 0$$
$$\frac{\partial h_t}{\partial Var^\theta[x_{t+1}|X_t]} \geq 0 \Leftrightarrow \gamma \leq \mathbb{E}^\theta[x_{t+1}|X_t]$$



(a) Density under a shift in the first moment



(b)Probability of default under a shift in the first moment

Figure 4: Default probability under changes in the moments of the conditional distribution $f(x_{t+1}|X_t)$

The propositions states that improvement in macroeconomic expectations leads to a decrease in the probability of default. While the proposition is formally proven in the Appendix, Figure 4 provides the intuition behind it. As the mean of the distribution of $x_{t+1}|X_t$ shifts to the right (Panel (a)), a lower mass of the distribution is below the risk of the entrepreneur γ. As a result, the adverse states in which the entrepreneur would default become less likely. Panel (b) illustrates that the default probability falls below the one under the initial distribution. This illustrates that, as the mean of the distribution increases, the expected default hazard falls.

4.3 The link between macroeconomic expectations and lending

Having already established the link between macroeconomic expectations and the probability of default, I now present the impact of changes in macroeconomic expectations on lending, welfare, and the bank failure risk. To start, I study this relationship under rational expectations. I consider rational expectations under precise macroeconomic information as a benchmark, as this is the usual assumption in the literature, and it underpins the expected effects of the introduction of CECL and IFRS9.

Proposition 5. *Under rational expectations, both in the constrained and unconstrained case, the following holds:*

$$\begin{aligned} &\textit{Total lending:} && \frac{\partial K_t}{\partial \mathbb{E}^{\theta}[x_{t+1}|X_t]} \geq 0 \\ &\textit{Total output:} && \frac{\partial Y_t}{\partial \mathbb{E}^{\theta}[x_{t+1}|X_t]} \geq 0 \\ &\textit{Total expected surplus:} && \frac{\partial S_t}{\partial \mathbb{E}^{\theta}[x_{t+1}|X_t]} \geq 0 \end{aligned}$$

The relationship between macroeconomic expectations and stability of the bank is non-monotonic:

$$\textit{Probability of bank failure:} \qquad \frac{\partial P_t}{\partial \mathbb{E}^{\theta}[x_{t+1}|X_t]} \leq 0, \; \textit{when } E_t + r_{1t}K_{1t} < K_{2t}$$

Economic activity generated by the risk-free project does not depend on the macroeconomic state, and hence does not depend on expectations. Expectations shape the activity of the risky entrepreneur, and through that effect shape overall lending, output, and surplus in the economy. As expectations improve, the expected default probability of the risky entrepreneur falls, which allows for the bank to offer a lower interest rate for the project and originate a higher amount of loan. This translates into a larger investment, larger output, and in addition to improving the prospective success of the project, it also generates higher expected surplus in the economy. The effect on the probability of bank failure is non-monotonic, however. When the default probability is high (more precisely, when the default hazard exceeds the threshold h^* such that $K_{2t}(h^*) = E_t + r_{1t}K_{1t}$), lending to the risky entrepreneur is so limited that, even if it defaults, the bank will be able to cover the loss and repay its deposits. The probability of bank failure in this case is 0 and remains 0, as long as the probability of default remains in that region. Once expectations improve to the extent that risky loan origination becomes substantial (above $E_t + r_{1t}K_{1t}$), the risk of bank failure jumps up. After this, as expectations further improve, the probability of bank failure falls, driven by the falling probability of default.

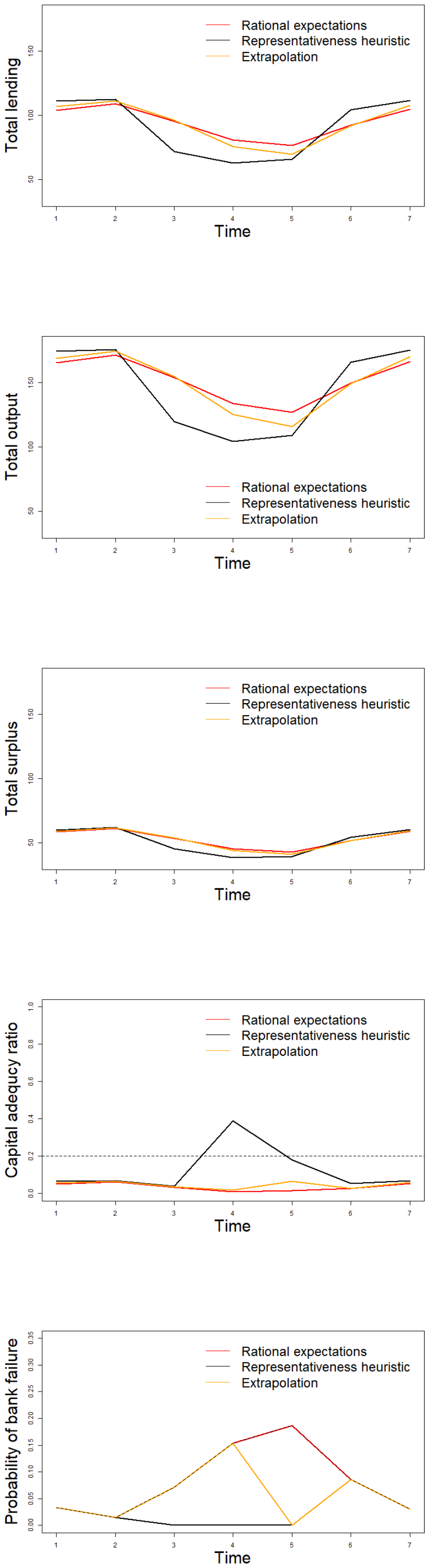
Proposition 5 also shows that, under rational expectations, optimal bank lending and economic output co-vary with the macroeconomic state. More generally, even when banks consider all available current macroeconomic information, when they observe the current macroeconomic state without any noise, when they are forward-looking and correctly evaluate the distribution of possible future macroeconomic states, and when there is no moral hazard problem affecting the bank's incentives, we obtain that optimal lending behavior is cyclical. This stems from the fact that the macroeconomic conditions are persistent, making the current macroeconomic state informative about the expected future returns.

I now explore how differences in the expectation formation process affect lending, welfare, and bank stability. In particular, I study how deviating from rational expectations to expectations based on the representativeness heuristic affects lending and all other variables of interest. This can highlight, perhaps, unintended consequences of forward-looking accounting on bank lending.

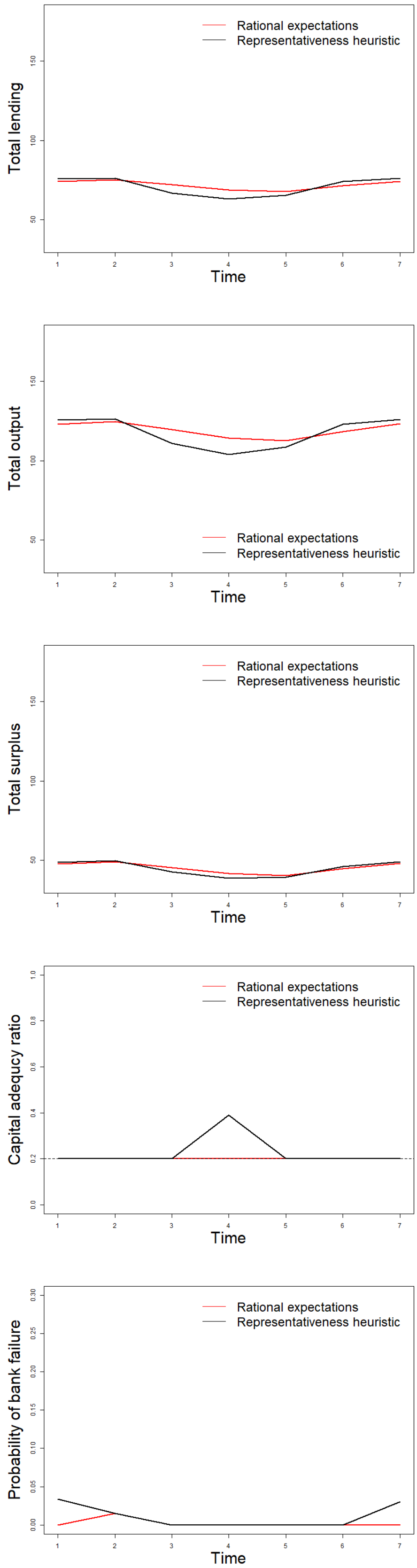
Based on the macroeoconomic evolution in Figure 3, Figure 5 illustrates how expectations affect lending and all related indicators along the economic cycle. As we see, when macroeconomic expectations are based on the representativeness heuristic, bank lending and output are more pro-cyclical than those based on rational expectations. In that sense, we can say that the representativeness heuristic causes excessive procyclicality.

Under the representativeness heuristic, good news leads to optimism, which leads to more extensive loan origination and larger output in the economy. Nevertheless, because the bank underestimates the default risk when using the representativeness heuristic, $h_t^{RE} > h_t^{RH}$, the risk exposure of the bank is higher than rational, $h_t^{RH}K_{2t}^{RH} < h_t^{RE}K_{2t}^{RH}$, and the surplus is overestimated $(1 - h_{2t}^{RE})K_{2t}^{RH} < (1 - h_{2t}^{RH})K_{2t}^{RH}$. As a result, the probability of bank failure is higher. Notice that the deviation between the rational and biased expectations gives rise to an expected loss overhang of $(h_t^{RE} - h_t^{RH})K_{2t}^{RH}$ based on the definition of unreported expected losses by Bushman and Williams (2015).

In the case of bad news, overreaction yields too gloomy expectations for the borrower's repayment probability, causing banks to shrink loan supply below the rational level. As a



(a) Unconstrained case



(b) Constrained case

Figure 5: Banking sector indicators under changes in the expectation $E^{\theta}[x_{t+1}|X_t]$
Simulation under the following parameters: $\rho = 0.7$, $\alpha = 0.8$, $\xi = 0.2$, $\sigma_u = 0.5$, $E_t = 4$, $\gamma = -0.7$

result, output in the economy falls too much, and too much surplus is forgone versus behavior under rational expectations. This precautionary behavior lowers the probability of bank failure.

When agents are less capable of retrieving the probabilities of unrepresentative states, meaning that the reliance on the representativeness heuristic is higher, the overreaction in lending to macroeconomic shocks is stronger.

These results suggest that reliance on forecasts when they are subject to overreaction to news leads to excessive lending in good times, which may foster a positive output gap at the cost of accumulating high risk in the banking sector. In contrast, overreaction to news leads to excessive precautionary behavior under bad news, which may lead to an amplification of the bad news by excessively shrinking production and causing a significant loss of welfare.

4.4 Interaction of expectations and the minimum capital requirement

The presence of a capital requirement does not have any impact on the bank's lending decision when the bank has enough equity to provide the optimal level of loans. Therefore, the effect of overreaction to economic news is not impacted.

Figure 5 presents a simulation of the key variables for a bank whose equity is below the capital adequacy minimum. Panel (a) presents the results when the bank is not regulated to maintain a capital adequacy ratio above the minimum threshold and is allowed to originate loans as large ad desired. Panel (b) presents the results when the same bank is subject to the forward-looking provisioning regime, which stipulates that the capital adequacy ratio should exceed the minimum threshold, which implies that the bank may need to originate loans in smaller amounts. As we can see, when the bank is constrained by the capital requirement, the requirement tends to limit loan origination under good news, and in that sense it limits the effect of the representativeness heuristic. Under bad news, loan origination falls and the capital adequacy ratio increases, making the capital requirement slack. Strong pessimism in bad times stemming from the representativeness heuristic, limits the bank exposure to risk and reduces the probability of bank failure. However, being slack, the capital requirement has no bite in restricting the procyclical reaction of bank lending in the case downturns, which is too strong compared to precautionary but rational reaction. The welfare cost stemming from the forecasting bias can be substantial. This finding supports the concerns expressed by banks in the debate around CECL that the forward-looking regime can exacerbate procyclicality, especially in recessions.

5 Comparison of loan origination under incurred loss and forward-looking provisioning

I consider two aspects in which the incurred loss and forward-looking provisioning regimes differ; namely, the timing of provisioning and the process used in expectation formation about credit losses. First, under the incurred loss framework, no loan loss provisions are accounted for at the time of loan origination, t. This resembles the property that, in an incurred loss provisioning system, loan loss provisions are driven by non-performing loans when there is evidence that losses are likely to occur. In contrast, under CECL, expected credit losses are accounted for more timely, based on reasonable and supportable forecasts of future economic conditions.

5.1 Timely provisioning

Under the incurred credit loss method, the bank does not recognize losses at the time of loan origination;[14] hence, it is subject to the following constraint: $\frac{E_t}{K_{2t}} \geq \xi$. To compare, the constraint under CECL is: $\frac{E_t - h_t K_{2t}}{(1-h_t)K_{2t}} \geq \xi$. Apart from the capital adequacy constraint, the bank problem is equivalent under both accounting methods.

Table 4 summarizes the amount of loan origination under the two provisioning methods. We can make the following observations. When the bank is not constrained by the capital requirement, loan origination is the same under the two regimes. This is expected, as the difference between the accounting regimes is that they affect the timing of when loan loss provisions are accounted for, and therefore affects the timing of the potential violations of the capital requirement. However, if the bank has enough equity not to worry about the capital requirement, then the timing of LLP does not matter. Therefore, when the bank has enough equity, the provisioning method does not impact bank lending, nor any of the bank or economy-level indicators presented in Section 3.4.1. In particular, if the bank is subject to behavior biases in its forecasts, the capital requirement does nothing to limit its overreaction to news.

On the other hand, when banks are constrained by the capital adequacy ratio, the accounting regime affects the amount of loan origination. Loan origination under the forward-looking regime coincides with that under the incurred loss regime only when the loan is risk-free, i.e. when $h_t \equiv \mathbb{E}[\mathbb{1}\{x_{t+1} < \gamma\}|X_t] = 0$. However, in the more realistic case where the loan default probability is non-zero, loan origination under the forward-looking regime is smaller. This

[14] I assume that the bank would not originate the loan if a loss is likely at this time, as it will not be profitable to originate the loan on expectation.

Table 4: Lending to the risky borrower under the two accounting regimes

Accounting framework	Constrained	Unconstrained	Constrained when
Incurred loss	$K_{2t} = \frac{E_t}{\xi}$	$K_{2t} = \left(\frac{1}{\alpha^2 A}\frac{1}{1-h_{2t}}\right)^{\frac{1}{\alpha-1}}$	$\alpha^2 A(1-h_{2t})\left(\frac{\xi}{E_t}\right)^{1-\alpha} > 1$
Forward-looking	$K_{2t} = \frac{E_t}{\xi + h_{2t}(1-\xi)}$	$K_{2t} = \left(\frac{1}{\alpha^2 A}\frac{1}{1-h_{2t}}\right)^{\frac{1}{\alpha-1}}$	$\alpha^2 A(1-h_{2t})\left(\frac{\xi+(1-\xi)h_{2t}}{E}\right)^{1-\alpha} > 1$

result echoes the argument expressed by some banks and politicians that CECL can increase the cost and lower the availability of credit. Overall, the bank is more conservative under the forward-looking provisioning method. This can limit the amount of excessive lending under good news, but it can exacerbate the problem of excessive pessimism when bad macroeconomic news arrives.[15] In other words, by stimulating precautionary behavior, the forward-looking regime can better limit pro-cyclical lending in good times, but also limit lending and exacerbate the loss in welfare in bad times.

Furthermore, when banks are constrained by the capital adequacy requirement, loan origination under the forward-looking regime depends on expectations about the macroeconomy through the default probability of the risky loan h_t. In contrast, loan origination under the incurred loss is only determined by the minimum capital requirement, which is assumed to be constant. Hence, under the incurred loss provisioning, loan origination is not affected by the expected macroeconomic conditions. This distinction highlights an intended consequence of using the forward-looking regime, as the name suggests: making bank behavior more sensitive to expectations of the economy rather than focusing on the current state alone. This, however, also shows that the forward-looking regime opens room for more sensitivity of bank behavior to the properties of the forecasts, when the bank is constrained by the capital requirement.

Lastly, note that that the conditions of when the bank is constrained by the capital require-

[15] Notice that under bad news, and more generally under bad macroeconomic states, the optimal level of lending to the risky entrepreneur is small. In this case, the capital adequacy ratio increases and the capital constraint may become slack. Therefore, the capital constraint only matters under positive macroeconomic states and under less severe macroeconomic contractions, but it does not have a bite when the macro state is so low that banks optimally shrink their risky lending.

ment under the forward-looking and incurred loss provisioning methods are the following:

$$C^{FL} \equiv \alpha^2 A(1-h_t)\left(\frac{\xi+(1-\xi)h_t}{E_t}\right)^{1-\alpha} - 1 > 0$$
$$C^{IL} \equiv \alpha^2 A(1-h_t)\left(\frac{\xi}{E_t}\right)^{1-\alpha} - 1 > 0$$

Also note that $C^{FL} \geq C^{IL}$. This expression shows that whenever the bank problem is constrained under the incurred loss regime, it is also constrained under the forward-looking one. In other words, IFRS 9/CECL makes the capital requirement more stringent. By this, forward-looking provisioning is more effective in limiting excessive lending in good times, when macroeconomic expectations can be too optimistic.

5.2 Change in the expectation formation process

One method of estimating expected credit losses which is commonly used in practice and also suggested in the literature (Harris et al., 2018), is to use a constant ratio between charge-offs and total loan amount. In my framework, this method is equivalent to assuming the default rate at $t+1$ will remain unchanged from the one observed at the current date t, which is consistent with the following macroeconomic expectation formation process, which I call extrapolation:

$$E^{extr}[x_{t+1}|x_t] = x_t$$

This method is subject to criticism, which was commonly expressed in relation to the incurred credit loss regime, according to which banks relied too much on current and past information, and did not pay enough attention to expectations about the future. To overcome this criticism, CECL and IFRS9 explicitly state that estimates about expected credit losses should reflect reasonable and supportable forecasts of future economic conditions. This stipulation might change the way expectations are formed: from relying on simple extrapolation to relying on macroeconomic forecast. Therefore, it is interesting the study the implications for lending from such a shift, which I discuss in this section.

Suppose the state is positive and good news have arrived. Rational expectations imply that $E[x_{t+1}] = \rho x_t$. In this case, if banks are using extrapolation to predict the probability of default, they are overly optimistic: $E^{extr}[x_{t+1}|X_t] = x_t > E^{RE}[x_{t+1}|X_t] = \rho x_t$. In this case, banks would be underestimating the probability of default, and lending excessively to the risky entrepreneur.

It would be optimal to bring the expectations closer to the rational expectations. However, if banks become more reliant on macroeconomic forecast, which are subject to the representativeness bias, there are parameters under which $E^{RH}[x_{t+1}|X_t] > E^{extr}[x_{t+1}|X_t] > E^{RE}[x_{t+1}|X_t]$. In particular this is the case when $\rho\theta u_t > (1-\rho)x_t$, i.e. when the economy is very persistent (ρ is high) and subject to high overreaction (θ is high). Bordalo et al. (2020) find that the diagnostic parameter θ in around 0.5 on average over 20 macroeconomic variables. Therefore, if x_t fluctuates around the expected value of 0 and $\rho > 0$, whenever there is a positive macroeconomic shock, the expectation under the representativeness heuristic will exceed the extrapolation and the rational one. Under negative shocks, overreaction can make expectations worse that a simple extrapolation and worse than a rational forecast: $E^{RH}[x_{t+1}|X_t] < E^{extr}[x_{t+1}|X_t] < E^{RE}[x_{t+1}|X_t]$. This shows that moving to a more forward-looking framework can exacerbate the procyclicality of the banking sector, even if the benchmark is not the perfect Bayesian forecasting, but the simple extrapolation benchmark.

6 Implications for banking regulation

Up to now I have derived how the different expectation formation processes affect provisions, lending, and the aggregate economic variables, taking the capital constraint as exogenously given and constant over time. In this section, I now study the question of what is the optimal capital requirement under the different expectation formation processes. In particular, what are the implications for regulatory capital, if expectations are forward-looking but subject to overreaction to news?

In this section, I simplify the model by assuming perfect competition. Although monopolistic competition may be a more realistic representation of the banking sector (as discussed earlier), it leads to a welfare loss due to banks' market power. This friction can interact with biases in expectations, as overly optimistic banks may lend more than they would under rational expectations. This can lead to an increase in welfare. To focus solely on the impact of expectations on welfare, I will concentrate on the case of perfect competition.

Suppose there is an infinite pool of entrepreneurs, who are subject to macroeconomic and idiosyncratic risk. Entrepreneur i defaults at time $t+1$ if $x_{t+1} + e_{i,t+1} \leq \gamma$, where x_{t+1} is the macroeconomic state at $t+1$ and $e_{i,t+1} \sim N(0, \sigma_e^2)$ is i.i.d. across individuals and over time and represents an individual-specific shock. Notice that the default probability $h_{i,t}^{\theta} = h_t^{\theta}$ is the same across individuals.[16] Similarly to before, each entrepreneur's demand is: $K_t = \left(\frac{1+r_t}{\alpha A}\right)^{-\frac{1}{1-\alpha}}$

Due to perfect competition, the bank sets the interest rate for each potential loan so that it breaks even:

$$(1 - h_t^\theta) r_t K_t - h_t^\theta K_t = 0$$

Therefore, the interest rate is set according to $r_t = \frac{h_t^\theta}{1-h_t^\theta}$ and the the corresponding lending amount is $K_t = \left(\alpha A(1 - h_t^\theta)\right)^{\frac{1}{1-\alpha}}$.

The bank originates N_t loans of size K_t until the capital constraint becomes binding:

$$\frac{E_t - h_t^\theta N_t K_t}{(1 - h_t^\theta) N_t K_t} = \xi_t$$

I now turn attention to the regulator's problem, who seeks to maximize the expected surplus in the economy by setting the minimum capital requirement, ξ_t. It is worth noting that the minimum capital requirement can vary over time. I assume that there are externalities associated with bank failure, which in turn motivates the need for banking regulation. I assume that there are externalities stemming from bank failure, which is a motivation for the need for banking regulation. I am not explicitly modelling the externalities. I assume that bank failure leads to a welfare loss to the economy, on top of the loss generated by the bank, but I am agnostic about the type of cost or mechanism bank failure gives rise to this cost. For example, we can think of the externality as a cost due to bank failure translating into lack of confidence in the banking system.

Furthermore, it is worth noting that my approach to addressing the regulator's problem is primarily theoretical in nature. Specifically, I examine what the optimal capital requirement would be if the expected surplus is derived using rational expectations. It is important to emphasize, however, that I do not delve into the analysis of potential biases that may arise in the regulator's expectations, which are likely to be present in the real-world.

[16] $h_i^\theta(x_t, u_t) = \int_{-\infty}^{\gamma} \phi\left(\frac{x_{t+1} - \mathbb{E}^\theta[x_{t+1}|x_t, u_t]}{\sigma_x + \sigma_e}\right) d(x_{t+1})$, where $\phi(.)$ denotes the density of standard normal distribution.

The expected surplus in the economy is:[17]

$$\mathbb{E}[S|x_{t-1}, u_t] = (1-h_t)N_tAK_t^\alpha - N_tK_t - \frac{ch_t(N_tK_t)^2}{2}$$

The first term represents the expected output in the economy in case production is realized on the market and does not perish with probability $1-h_t$. The second term is the sunk investment cost. Lastly, the third terms represents the cost of bank failure, which is assumed to be an increasing function of the total loan origination amount, where c is a fixed constant.

The regulator's problem can be simplified to the following:

$$\max_{\xi_t} \left(\frac{1-h_t}{\alpha(1-h_t^\theta)} - 1\right) N_tK_t - \frac{ch(N_tK_t)^2}{2}$$

Table 5 summarizes the optimal capital adequacy minimum and the corresponding surplus in the economy.[18] The first column presents the benchmark case, in which expectations are rational, i.e. when $h_t^\theta = h$.The second column presents the results when expectations can be subject to the representativeness heuristic, or extrapolation.

Notice that $\frac{\partial \xi^{RE}}{\partial h} > 0$. When expectations are rational, the optimal capital requirement depends on the underlying default hazard in the economy, h_t. The higher this risk, the more stringent the minimum capital adequacy ration. When expectations deviate from rationality, the capital requirement depends on both the underlying default risk, h_t, but also on the perceived risk by the banks, h_t^θ. It is still the case that $\frac{\partial \xi^{RH/extr}}{\partial h_t} > 0$, i.e. required capital is increasing

[17]Throughout the analysis I assume that $\frac{\alpha}{1-\alpha} cE_t > 1$. This is equivalent to assuming $\frac{cE_t}{h_t+\xi(1-h_t)} > \frac{1-\alpha}{\alpha}$ when $h_t = 1$; in other words, when the default hazard is 100%, the marginal cost from lowering the capital requirement exceeds the marginal benefit.

[18]The table only presents the optimal capital adequacy minimum when the solution is interior. More generally, when expectations are rational:

$$\text{If interior solution, } \xi_t^* = \frac{h_t}{1-h_t}\left[\frac{\alpha c E_t}{1-\alpha} - 1\right]$$
$$\text{If } \frac{1-\alpha}{\alpha} > \frac{ch_tE}{h_t + \underline{\xi}(1-h_t)}, \xi = \underline{\xi}$$
$$\text{If } \frac{1-\alpha}{\alpha} < \frac{ch_tE}{h_t + \bar{\xi}(1-h_t)}, \xi = \bar{\xi}$$

Similarly, when expectations are allowed to deviate from rationality:

$$\text{If interior solution, } \xi_t^* = \frac{1}{1-h_t^\theta}\left[\frac{\alpha ch_tE_t(1-h_t^\theta)}{1-h_t-\alpha(1-h_t^\theta)} - h_t^\theta\right]$$
$$\text{If } \frac{1-h_t}{\alpha(1-h_t^\theta)} - 1 > \frac{ch_tE}{h_t^\theta + \underline{\xi}(1-h_t^\theta)}, \xi = \underline{\xi}$$
$$\text{If } \frac{1-h_t}{\alpha(1-h_t^\theta)} - 1 < \frac{ch_tE}{h_t^\theta + \bar{\xi}(1-h_t^\theta)}, \xi = \bar{\xi}$$

Table 5: Optimal capital requirement and corresponding surplus under different expectation formation processes

	Rational expectations	RH/Extrapolation
Capital adequacy	$\xi_t^{RE} = \frac{h_t}{1-h_t}\left(\frac{\alpha c E}{1-\alpha} - 1\right)$	$\xi_t^{RH/extr} = \frac{1}{1-h_t^\theta}\left[\frac{\alpha c h_t E_t(1-h_t^\theta)}{1-h_t-\alpha(1-h_t^\theta)} - h_t^\theta\right]$
Surplus	$S_t^{RE} = \frac{1}{2ch_t}\frac{(1-\alpha)^2}{\alpha^2}$	$S_t^{RH/extr} = \frac{1}{2ch}\left[\frac{1-h_t-\alpha(1-h_t^\theta)}{\alpha(1-h_t^\theta)}\right]^2$

in the default risk, but also $\frac{\partial \xi^{RH/extr}}{\partial h^\theta} < 0$. The latter means that, holding the underlying risk h_t in the economy constant, when banks are more pessimistic (in the sense that h_t^θ is higher), the regulator should lower the capital requirement. In a sense, optimal policy undoes the bias in bank expectations.

This section shows that understanding the properties of forward-looking provisions is important, because they affect the users of information. For regulators, the properties of provision numbers matter, because they affect regulatory capital. Hence, this section highlights the interdependence of accounting standards and bank regulation.

6.1 Variation over time

In this section I explore how ξ_t^{RE}, ξ_t^{RH} and ξ_t^{extr} vary over time.

I now use the derived optimal capital constraints in Table 5, as well as the expectation formation processes presented in Table 3, in order to approximate the optimal capital constraints around the expected value of the macroeconomic state and the macro shock state (x_{t-1}, u_t), which is $(0,0)$. I obtain the how the following expressions:

$$\xi^{RE}(x_{t-1}, u_t) \approx A_0 + (A_1 - A_2)\bar{\Phi}(\gamma)\rho^2 x_{t-1} + (A_1 - A_2)\bar{\Phi}(\gamma)\rho u_t$$

$$\xi^{RH}(x_{t-1}, u_t) \approx A_0 + (A_1 - A_2)\bar{\Phi}(\gamma)\rho^2 x_{t-1} + (A_1 - A_2 - \theta A_2)\bar{\Phi}(\gamma)\rho u_t$$

$$\xi^{extr}(x_{t-1}, u_t) \approx A_0 + \left(A_1 - \frac{1}{\rho}A_2\right)\bar{\Phi}(\gamma)\rho^2 x_{t-1} + \left(A_1 - \frac{1}{\rho}A_2\right)\bar{\Phi}(\gamma)\rho u_t$$

where A_1 and A_2 are positive constants, and $A_1 + A_2 > 0$

For high overreaction to news $\theta > \bar{\theta}$, where $\bar{\theta}$ is a fixed threshold:

- $Var(\xi_t^{RH}) > Var(\xi_t^{RE})$

For high persistence in the economy $\rho > \bar{\rho}$, where $\bar{\rho}$ is a fixed threshold:

- $Var(\xi_t^{RE}) > Var(\xi_t^{extr})$

These results imply that moving from the extrapolation method in provisioning to rational expectations, highlights the stronger need for time-varying capital constraint. This is further exacerbated if expectations are forward-looking and subject to a strong representativeness bias.

Behavior biases in forecasts highlight the time-variation of the problem, and point to the potential usefulness of time-varying capital-constraint (in line with Stein (2021))

7 Conclusion

In this paper, I study how the CECL/IFRS 9 affects bank lending when macroeconomic expectations are subject to behavior biases. In particular, I allow for expectations to be based on Kahneman and Tversky's (1972) representativeness heuristic, formalized in the diagnostic expectations framework of Gennaioli and Shleifer (2010) and Bordalo et al. (2020). Empirical studies have shown that imprecise information and the representativeness heuristic are related to the observed deviations from FIRE in macroeconomic forecasts in numerous settings. These are, by far, not the only frictions that can make forecasts deviate from the full information rational expectations model. However, it is beyond the scope of this paper to explore all factors that cause forecasts to depart from FIRE. Instead, focusing on the representativeness heuristic, which is micro-founded and empirically documented to play a role in many settings, I explore potential unintended consequences of bank lending once the forward-looking provisioning method is implemented.

I build a simple model of bank lending, in which the bank is subject to a capital adequacy constraint. A key feature of the model is that the return on loans depends on future macroeconomic conditions, which makes macroeconomic expectations important for provisioning and bank lending. Under the representativeness heuristic, good news leads to excessive optimism, which drives extensive loan origination and strong economic activity. Nevertheless, because the bank underestimates the default risk when using the representativeness heuristic, risk exposure of the bank is higher than rational, which raises the probability of bank failure. In contrast, overreaction to bad news leads to excessive precautionary behavior in downturns, which can cause a significant loss of welfare compared to the rational benchmark. These results suggest that reliance on forecasts, when forecasts are subject to biases, can lead to undesirable outcomes. The outcomes can be inferior even when compared to decision-making based on simple extrapolation of the current data, in case the economy is persistent, and when macroeconomic forecasts exhibit strong overreaction to news. The capital adequacy requirement can constrain the effect of the representativeness heuristic amid good news, but cannot limit the excessive pessimism in case of downturns.

More generally, my analysis suggests that procyclical behavior in bank lending might stem from the properties of the forecasts rather than on banks not being forward-looking enough. It contributes to the accounting literature which cautions that accurately predicting expected credit losses is difficult, and highlights one channel through which imperfect forecasts drive inefficient outcomes.

Last, I derive the optimal capital adequacy requirement, given the impact of macroeconomic expectations on provisions. I find that the optimal capital requirement must react to the underlying default risk in the economy and also undo the effects of the biases in banks' expectations. Furthermore, my analysis suggests that switching to a forward-looking regime in provisions, as well as the potential for overreaction to news, both imply that the optimal capital constraint varies more considerably over time.

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Appendix A

Allowance for Credit Losses

The allowance for credit losses includes the allowance for loan and lease losses and the reserve for unfunded lending commitments. Our process for determining the allowance for credit losses is discussed in *Note 1 – Summary of Significant Accounting Principles* and *Note 5 – Outstanding Loans and Leases and Allowance for Credit Losses* to the Consolidated Financial Statements.

The determination of the allowance for credit losses is based on numerous estimates and assumptions, which require

a high degree of judgment and are often interrelated. A critical judgment in the process is the weighting of our forward-looking macroeconomic scenarios that are incorporated into our quantitative models. As any one economic outlook is inherently uncertain, the Corporation uses multiple macroeconomic scenarios in its ECL calculation, which have included a baseline scenario derived from consensus estimates, an adverse scenario reflecting an extended moderate recession, a downside scenario reflecting persistent inflation and interest rates above the baseline scenario, a tail risk scenario similar to the severely adverse scenario used in stress testing and an upside scenario that considers the potential for improvement above the baseline scenario. The overall economic outlook is weighted 95 percent towards a recessionary environment in 2023, with continued inflationary pressures leading to lower gross domestic product (GDP) and higher unemployment rate expectations as compared to the prior year. Generally, as the consensus estimates improve or deteriorate, the allowance for credit losses will change in a similar direction. There are multiple variables that drive the macroeconomic scenarios with the key variables including, but not limited to, U.S. GDP and unemployment rates. As of December 31, 2021, the weighted macroeconomic outlook for the U.S. average unemployment rate was forecasted at 5.2 percent, 4.7 percent and 4.3 percent in the fourth quarters of 2022, 2023 and 2024, respectively, and the weighted macroeconomic outlook for U.S. GDP was forecasted to grow at 2.1 percent, 1.9 percent and 1.9 percent year-over-year in the fourth quarters of 2022, 2023 and 2024, respectively. As of December 31, 2022, the latest consensus estimates for the U.S. average unemployment rate for the fourth quarter of 2022 was 3.7 percent and U.S. GDP was forecasted to grow 0.4 percent year-over-year in the fourth quarter of 2022, reflecting a tighter labor market and depressed growth expectations compared to our macroeconomic outlook as of December 31, 2021, and were factored into our allowance for credit losses estimate as of December 31, 2022. In addition, as of December 31, 2022, the weighted macroeconomic outlook for the U.S. average unemployment rate was forecasted at 5.6 percent and 5.0 percent in the fourth quarters of 2023 and 2024, and the weighted macroeconomic outlook for U.S. GDP was forecasted to contract 0.4 percent and grow 1.2 percent year-over-year in the fourth quarters of 2023 and 2024.

Figure 6: Excerpt from Form 10K of Bank of America Corporation for the fiscal year ended Dec 31, 2021

Allowance for Credit Losses

The allowance for credit losses includes both the allowance for loan losses and the allowance for credit losses on lending-related commitments. As a percentage of total loans, the allowance for credit losses was 1.26 percent at December 31, 2021, compared to 1.90 percent at December 31, 2020. Excluding PPP loans, which are guaranteed by the Small Business Administration, the allowance for credit losses was 1.27 percent of total loans at December 31, 2021, compared to 2.03 percent at December 31, 2020. The allowance for credit losses covered 2.3 times and 2.8 times total nonperforming loans at December 31, 2021 and December 31, 2020, respectively.

The allowance for credit losses decreased by $374 million from $992 million at December 31, 2020 to $618 million at December 31, 2021, primarily reflecting strong credit quality, a sustained recovery by the U.S. economy from the COVID-19 pandemic and benefits of mitigating actions by the U.S. government. In addition to $2.8 trillion of stimulus spending approved by Congress between December 2020 and March 2021, the rollout of the COVID vaccine, strong business spending and improved labor markets contributed to an overall improved economic outlook. However, there continues to be uncertainty related to the impact of emerging COVID-19 variants and vaccine efficacy, supply chain constraints, future monetary and fiscal support and inflationary pressures.

These factors shaped the 2-year reasonable and supportable forecast used by the Corporation in its CECL modeled estimate at December 31, 2021. The U.S. economy is expected to grow at a moderate pace through the first half of 2022 before normalizing into historical growth rates. Forecasts for other key economic variables, including the unemployment rate, are generally in line with Gross Domestic Product (GDP) projections. Oil prices are expected to decrease from current elevated levels. Interest rates are expected to increase, reflecting the Federal Reserve's revised expectations, while corporate bond spreads reflect normalized default risk. The following table summarizes select economic variables representative of the economic forecasts used to develop the allowance for credit losses estimate at December 31, 2021.

Economic Variable	Base Forecast
Real GDP growth	Gradually normalizes to a long-term growth rate of 2 to 3 percent by third quarter 2022.
Unemployment rate	Current levels decrease to 4 percent by second quarter 2022, remaining between 3.5 percent and 4 percent through the remainder of the forecast period.
Corporate BBB bond to 10-year Treasury bond spreads	Spreads remain below 2 percent throughout the forecast period.
Oil Prices	Prices gradually decline from current levels to $62 by second quarter 2023.

Figure 7: Excerpt from Form 10K of Comerica Incorporated for the fiscal year ended Dec 31, 2021

ALLOWANCE FOR CREDIT LOSSES

The Firm's allowance for credit losses represents management's estimate of expected credit losses over the remaining expected life of the Firm's financial assets measured at amortized cost and certain off-balance sheet lending-related commitments. The Firm's allowance for credit losses comprises:

- the allowance for loan losses, which covers the Firm's retained loan portfolios (scored and risk-rated) and is presented separately on the Consolidated balance sheets,
- the allowance for lending-related commitments, which is reflected in accounts payable and other liabilities on the Consolidated balance sheets, and
- the allowance for credit losses on investment securities, which is reflected in investment securities on the Consolidated balance sheets.

Discussion of changes in the allowance

The allowance for credit losses as of December 31, 2022 was $22.2 billion, reflecting a net addition of $3.5 billion from December 31, 2021, consisting of:

- $2.3 billion in wholesale, driven by deterioration in the Firm's macroeconomic outlook and loan growth, predominantly in CB and CIB, and
- $1.2 billion in consumer, predominantly driven by Card Services, reflecting higher outstanding balances and deterioration in the Firm's macroeconomic outlook, partially offset by a reduction in the allowance related to a decrease in uncertainty associated with borrower behavior as the effects of the pandemic gradually recede.

Deterioration in the Firm's macroeconomic outlook included both updates to the central scenario in the fourth quarter of 2022, which now reflects a mild recession, as well as the impact of the increased weight placed on the adverse scenarios beginning in the first quarter of 2022 due to the effects associated with higher inflation, changes in monetary policy, and geopolitical risks, including the war in Ukraine.

The Firm's allowance for credit losses is estimated using a weighted average of five internally developed macroeconomic scenarios. The adverse scenarios incorporate more punitive macroeconomic factors than the central case assumptions provided in the table below, resulting in a weighted average U.S. unemployment rate peaking at 5.6% in the second quarter of 2024, and a 1.2% lower U.S. real GDP exiting the second quarter of 2024.

The Firm's central case assumptions reflected U.S. unemployment rates and U.S. real GDP as follows:

	Assumptions at December 31, 2022		
	2Q23	**4Q23**	**2Q24**
U.S. unemployment rate[(a)]	**3.8 %**	**4.3 %**	**5.0 %**
YoY growth in U.S. real GDP[(b)]	**1.5 %**	**0.4 %**	**— %**

	Assumptions at December 31, 2021		
	2Q22	4Q22	2Q23
U.S. unemployment rate[(a)]	4.2 %	4.0 %	3.9 %
YoY growth in U.S. real GDP[(b)]	3.1 %	2.8 %	2.1 %

(a) Reflects quarterly average of forecasted U.S. unemployment rate.

(b) The year over year growth in U.S. real GDP in the forecast horizon of the central scenario is calculated as the percentage change in U.S. real GDP levels from the prior year.

Subsequent changes to this forecast and related estimates will be reflected in the provision for credit losses in future periods.

Refer to Critical Accounting Estimates Used by the Firm on pages 149-152 for further information on the allowance for credit losses and related management judgments.
Refer to Consumer Credit Portfolio on pages 110-115, Wholesale Credit Portfolio on pages 116-126 for additional information on the consumer and wholesale credit portfolios.

Figure 8: Excerpt from Form 10K of JPMorgan Chase & Co for the fiscal year ended Dec 31, 2022

Forecast Model Inputs as of December 2022

When modeling expected credit losses, the firm employs a weighted, multi-scenario forecast, which includes baseline, adverse and favorable economic scenarios. As of December 2022, this multi-scenario forecast was weighted towards the baseline and adverse economic scenarios.

The table below presents the forecasted U.S. unemployment and U.S. GDP growth rates used in the baseline economic scenario of the forecast model.

	As of December 2022
U.S. unemployment rate	
Forecast for the quarter ended:	
June 2023	4.2 %
December 2023	4.6 %
June 2024	4.6 %
Growth in U.S. GDP	
Forecast for the year:	
2023	0.4 %
2024	1.3 %
2025	1.7 %

The adverse economic scenario of the forecast model reflects a global recession in 2023 and a more aggressive tightening of monetary policy by central banks, resulting in an economic contraction and rising unemployment rates. In this scenario, the U.S. unemployment rate peaks at approximately 7.4% during the first quarter of 2024 and the maximum decline in the quarterly U.S. GDP relative to the fourth quarter of 2022 is approximately 2.7%, which occurs during the fourth quarter of 2023.

In the table above:

- U.S. unemployment rate represents the rate forecasted as of the respective quarter-end.
- Growth in U.S. GDP represents the year-over-year growth rate forecasted for the respective years.
- While the U.S. unemployment and U.S. GDP growth rates are significant inputs to the forecast model, the model contemplates a variety of other inputs across a range of scenarios to provide a forecast of future economic conditions. Given the complex nature of the forecasting process, no single economic variable can be viewed in isolation and independently of other inputs.

Figure 9: Excerpt from the 10K of Goldman Sachs Group Inc for the fiscal year ended Dec 31, 2022

	Cluster 1 Rational expectations	Cluster 2 Overreaction to news
Total Assets (in log)	13.49	12.86
EBIT/Sales ratio	0.46	0.42
Share of banks listed on major exchange	0.80	0.83
Share of banks incorporated in the US	0.60	0.50

Table 6: Comparison between banks by cluster

Appendix B

Proof of Proposition 1

The macroeconomic process is the following: $x_t = \rho x_{t-1} + u_t$ and using forward substitution we obtain $x_{t+T} = \rho^T x_t + \sum_{i=1}^{T-1} \rho^i u_{t+T-1}$ for $T \geq 1$. Therefore, conditional on x_t, x_{t+T} is a sum of normally distributed random variables. Under precise information, conditioning on x_t and conditioning on S_t is equivalent in this case. Therefore, $x_{t+T}|S_t$ is normal. To characterize the mean and variance:

$$\begin{aligned}
\mathbb{E}[x_{t+T}|S_t] &= \mathbb{E}\left[\rho^T x_t + \sum_{i=1}^{T-1} \rho^i u_{t+T-1}|x_t\right] = \rho^T x_t \\
Var(x_{t+T}|S_t) &= Var\left(\rho^T x_t + \sum_{i=1}^{T-1} \rho^i u_{t+T-1}|x_t\right) \\
&= \sum_{i=1}^{T-1} \rho^i \sigma_u^2 = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2
\end{aligned}$$

Proof of Proposition 2

The distribution perturbed by the representativeness heuristic is:

$$f^\theta(x_{t+T}|x_t = \hat{x}_t) = f(x_{t+T}|x_t = \hat{x}_t)\left[\frac{f(x_{t+T}|x_t = \hat{x}_t)}{f(x_{t+T}|x_t = \rho\hat{x}_{t-1})}\right]^\theta \frac{1}{Z}$$

From Proposition 1 we know that $f(x_{t+T}|x_t = \hat{x}_t)$ is a normal distribution with:

$$\begin{aligned}
\mathbb{E}[x_{t+T}|x_t = \hat{x}_t] &= \rho^T \hat{x}_t \\
Var(x_{t+T}|x_t = \hat{x}_t) &= \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2
\end{aligned}$$

Similarly, $f(x_{t+T}|x_t = \rho\hat{x}_{t-1})$ is a normal distribution with:

$$\begin{aligned}
\mathbb{E}[x_{t+T}|x_t = \rho\hat{x}_{t-1}] &= \rho^{T+1}\hat{x}_{t-1} \\
Var(x_{t+T}|x_t = \rho\hat{x}_{t-1}) &= \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2
\end{aligned}$$

Denote $\mathbb{E}[x_{t+T}|x_t = \hat{x}_t] \equiv m_1$, $\mathbb{E}[x_{t+T}|x_t = \rho\hat{x}_{t-1}] \equiv m_2$ and $Var(x_{t+T}|x_t = \hat{x}_t) = Var(x_{t+T}|x_t = \rho\hat{x}_{t-1}) \equiv \sigma_T^2$.

The diagnostic distribution is then:

$$\begin{aligned}
f^\theta(x_{t+T}|x_t = \hat{x}_t) &= f(x_{t+T}|x_t = \hat{x}_t)\left[\frac{f(x_{t+T}|x_t = \hat{x}_t)}{f(x_{t+T}|x_t = \rho\hat{x}_{t-1})}\right]^\theta \frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)\left[\frac{\frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)}{\frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)}\right]^\theta \frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)\left[\frac{\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2}\right)}{\exp\left(-\frac{(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)}\right]^\theta \frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2} - \theta\frac{(x_{t+T}-m_1)^2}{2\sigma_T^2} + \theta\frac{(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)\frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(1+\theta)(x_{t+T}-m_1)^2 - \theta(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)\frac{1}{Z}
\end{aligned}$$

We can express the numerator in the exponential as:

$$\begin{aligned}
(1+\theta)(x_{t+T}-m_1)^2 - \theta(x_{t+T}-m_2)^2 &= (1+\theta)(x_{t+T}^2 - 2x_{t+T}m_1 + m_1^2) - \theta(x_{t+T}^2 - 2x_{t+T}m_2 + m_2^2) \\
&= x_{t+T}^2 + x_{t+T}(-2(1+\theta)m_1 + 2\theta m_2) + (1+\theta)m_1^2 - \theta m_2^2 \\
&= x_{t+T}^2 - 2x_{t+T}((1+\theta)m_1 - \theta m_2) + (1+\theta)m_1^2 - \theta m_2^2 \\
&= x_{t+T}^2 - 2x_{t+T}((1+\theta)m_1 - \theta m_2) + ((1+\theta)m_1 - \theta m_2)^2 \\
&- ((1+\theta)m_1 - \theta m_2)^2 + (1+\theta)m_1^2 - \theta m_2^2 \\
&= (x_{t+T} - ((1+\theta)m_1 - \theta m_2))^2 \\
&- \underbrace{((1+\theta)m_1 - \theta m_2)^2 + (1+\theta)m_1^2 - \theta m_2^2}_{c} \\
&= (x_{t+T} - ((1+\theta)m_1 - \theta m_2))^2 - c
\end{aligned}$$

Therefore, $f^\theta(x_{t+T}|x_t = \hat{x}_t)$ becomes:

$$\begin{aligned}
f^\theta(x_{t+T}|x_t = \hat{x}_t) &= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(1+\theta)(x_{t+T}-m_1)^2 - \theta(x_{t+T}-m_2)^2}{2\sigma_T^2}\right)\frac{1}{Z} \\
&= \frac{1}{\sqrt{2\pi\sigma_T^2}}\exp\left(-\frac{(x_{t+T} - ((1+\theta)m_1 - \theta m_2))^2}{2\sigma_T^2}\right)\frac{\exp(c/2\sigma_T^2)}{Z}
\end{aligned}$$

We can set $\exp(c/2\sigma_T^2) = Z$. As we see, the diagnostic distribution is also normal, with:

$$\begin{aligned}
\mathbb{E}^\theta[x_{t+T}|x_t = \hat{x}_t] &= (1+\theta)m_1 - \theta m_2 \\
&= (1+\theta)\mathbb{E}(x_{t+T}|x_t = \hat{x}_t) - \theta\mathbb{E}(x_{t+T}|x_t = \rho\hat{x}_{t-1}) \\
&= (1+\theta)\mathbb{E}(x_{t+T}|x_t = \hat{x}_t) - \theta\mathbb{E}(x_{t+T}|x_{t-1} = \hat{x}_{t-1}) \\
Var^\theta(x_{t+T}|x_t = \hat{x}_t) &= \sigma_T^2 = \frac{1-\rho^{2T}}{1-\rho^2}\sigma_u^2
\end{aligned}$$

In particular, when $T = 1$:

$$E^\theta[x_{t+1}|x_t] = (1+\theta)\mathbb{E}(x_{t+1}|x_t) - \theta\mathbb{E}(x_{t+1}|x_{t-1})$$

The bank's problem

Under the forward-looking regime, the Lagrangian is:

$$\mathcal{L}^{FL} = r_1K_1 + (1-h)(1+r_2)K_2 - hK_2 + \lambda^{FL}\left[E_t - (h + \xi(1-h))K_2\right]$$

Under the incurred loss regime, the Lagrangian is:

$$\mathcal{L}^{FL} = r_1K_1 + (1-h)(1+r_2)K_2 - hK_2 + \lambda^{IL}\left[E_t - \xi K_2\right]$$

where:

$$K_i = \left(\frac{1+r_i}{\alpha^2 A}\right)^{\frac{1}{\alpha-1}}, \; i = 1,2$$

Link between the capital adequacy constraint and default risk

Under the forward-looking regime, the capital adequacy constraint is binding when:

$$C^{FL} = \alpha^2 A(1-h)\left(\frac{\xi + (1-\xi)h}{E}\right)^{1-\alpha} - 1 > 0$$

Therefore:

$$\frac{dC^{FL}}{dh} = ((2-\alpha)(1-\xi) - 1 - (2-\alpha)(1-\xi)h)\left(\frac{\xi + (1-\xi)h}{E}\right)^{1-\alpha}\frac{1}{E}$$

When small default risk, i.e. $h < \frac{(2-\alpha)(1-\xi)-1}{(2-\alpha)(1-\xi)}$, $dC^{FL}/dh > 0$, i.e. the constraint can become biding. For large default risk, i.e. $h > \frac{(2-\alpha)(1-\xi)-1}{(2-\alpha)(1-\xi)}$, the constraint is less likely to hold (in the sense that it holds for a smaller set of parameter values). As the default hazard increases, the bank's optimal lending to the risky project falls, the capital adequacy rises, and the constraint is not binding. If $\frac{(2-\alpha)(1-\xi)-1}{(2-\alpha)(1-\xi)} < 0$ (i.e. when α is large and ξ is small), C^{FL} falls throughout.

Proof of Proposition 3

If the bank is not subject to a capital constraint:

$$
\begin{aligned}
K_2^{FB} &= \left(\frac{1}{\alpha^2 A(1-h)}\right)^{\frac{1}{\alpha-1}} \\
Y_2^{FB} &= AK_2^{FB\alpha} = \frac{1}{\alpha^2(1-h)}K_2^{FB} \\
S_2^{FB} &= (1-h)Y_2^{FB} - K_2 = \frac{1-\alpha^2}{\alpha^2}K_2^{FB}
\end{aligned}
$$

If the same bank is subject to a capital constraint and this constraint is binding:

$$
\begin{aligned}
K_2 &= \left(\frac{1+\lambda(\xi+(1-\xi)h)}{\alpha^2 A(1-h)}\right)^{\frac{1}{\alpha-1}} \\
Y_2 &= AK_2{}^{\alpha} = \frac{1+\lambda(\xi+(1-\xi)h)}{\alpha^2(1-h)}K_2 \\
S_2 &= (1-h)Y_2^{FB} - K_2 \\
&= \frac{1+\lambda(\xi+(1-\xi)h)-\alpha^2}{\alpha^2}\left(\frac{1+\lambda(\xi+(1-\xi)h)}{\alpha^2 A(1-h)}\right)^{\frac{1}{\alpha-1}} \\
&= \frac{1+\lambda(\xi+(1-\xi)h)-\alpha^2}{\alpha^2}(1+\lambda(\xi+(1-\xi)h)^{\frac{1}{\alpha-1}}K_2^{FB}
\end{aligned}
$$

We can show that the last expression is maximized when $\lambda(\xi+(1-\xi)h) = 0$ in which case it is equal to the surplus under the first best case.

Proof of Proposition 4

As shown in Propositions 1 and 2, the conditional distribution of $x_{t+2}|St$ is normal. For a normal distribution $N(\mu, \sigma^2)$ with density $f_X(x)$, let ϕ and Φ denote the probability and cumulative density functions of the standard normal distribution. In this case the repayment probability is:

$$\int_{\gamma}^{\infty} f_X(x)dx = 1 - \int_{-\infty}^{\gamma} f_X(x)dx$$

First, let us derive the derivative of the repayment probability with respect to the first moment of the distribution, μ:

$$\begin{aligned}
f_X(x) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \\
\frac{\partial f_X(x)}{\partial \mu} &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \frac{x-\mu}{\sigma^2}
\end{aligned}$$

Using Leibniz rule:

$$\frac{\partial \int_{-\infty}^{\gamma} f_X(x)dx}{\partial \mu} = \int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \frac{x-\mu}{\sigma^2}$$

Denote $z = \frac{x-\mu}{\sigma}$:

$$\begin{aligned}
&\int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \frac{x-\mu}{\sigma^2} dx = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\gamma} z \exp\left(-\frac{z^2}{2}\right) dz \\
&= \frac{1}{\sqrt{2\pi}\sigma} \left[-\exp\left(\frac{z^2}{2}\right) \Big|_{-\infty}^{\gamma} \right] = -\frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(\gamma-\mu)^2}{2\sigma^2}\right) \\
&= -\frac{1}{\sigma}\phi\left(\frac{\gamma-\mu}{\sigma}\right) \leq 0
\end{aligned}$$

Therefore, for the repayment probability we obtain:

$$\frac{\partial \int_{\gamma}^{\infty} f_X(x)dx}{\partial \mu} = \frac{1}{\sigma}\phi\left(\frac{\gamma-\mu}{\sigma}\right) \geq 0$$

Second, let us derive the derivative of the repayment probability with respect to σ:

$$\begin{aligned}
f_X(x) &= \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \\
\frac{\partial f_X(x)}{\partial \sigma} &= -\frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) + \frac{(x-\mu)^2}{\sqrt{2\pi}\sigma^4} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)
\end{aligned}$$

Using Leibniz rule:

$$\frac{\partial \int_{-\infty}^{\gamma} f_X(x)dx}{\partial \sigma} = -\int_{-\infty}^{\gamma} \frac{1}{\sqrt{2\pi}\sigma^2} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx + \int_{-\infty}^{\gamma} \frac{(x-\mu)^2}{\sqrt{2\pi}\sigma^4} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

Simplifying the first term:

$$-\int_{-\infty}^{\gamma}\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx=-\frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right)$$

Simplifying the second term, let $z\equiv\frac{x-\mu}{\sigma}$ and using integration by parts with $u=z, v=-\exp\left(-\frac{z^2}{2}\right), v'=z\exp\left(-\frac{z^2}{2}\right)$:

$$\begin{aligned}&\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\gamma}\frac{(x-\mu)^2}{\sigma^2}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\gamma}z^2\exp\left(-\frac{z^2}{2}\right)dz=\\&=\frac{1}{\sqrt{2\pi}\sigma}\left[-z\exp\left(-\frac{z^2}{2}\right)\Big|_{-\infty}^{\gamma}+\int_{-\infty}^{\gamma}\exp\left(-\frac{z^2}{2}\right)\right]\\&=-\frac{\gamma-\mu}{\sigma^2}\phi\left(-\frac{\gamma-\mu}{\sigma}\right)+\frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right)\end{aligned}$$

Combining all terms we obtain:

$$\begin{aligned}&\frac{\partial\int_{-\infty}^{\gamma}f_X(x)dx}{\partial\sigma}=-\frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right)-\frac{\gamma-\mu}{\sigma^2}\phi\left(\frac{\gamma-\mu}{\sigma}\right)+\frac{1}{\sigma}\Phi\left(\frac{\gamma-\mu}{\sigma}\right)\\&=-\frac{\gamma-\mu}{\sigma^2}\phi\left(\frac{\gamma-\mu}{\sigma}\right)\end{aligned}$$

Therefore, for the repayment probability we obtain:

$$\frac{\partial\int_{\gamma}^{\infty}f_X(x)dx}{\partial\sigma}=\frac{\gamma-\mu}{\sigma^2}\phi\left(\frac{\gamma-\mu}{\sigma}\right)<0\Leftrightarrow\gamma<\mu$$

Similarly, we can show that:

$$\frac{\partial\int_{\gamma}^{\infty}f_X(x)dx}{\partial\sigma^2}=\frac{1}{2}\frac{\gamma-\mu}{\sigma^3}\phi\left(\frac{\gamma-\mu}{\sigma}\right)<0\Leftrightarrow\gamma<\mu$$

Proof of Proposition 5

In the unconstrained case:

$$\begin{aligned}\frac{\partial K_2}{\partial E[x_{t+1}]}&=-\frac{K_2}{(1-\alpha)(1-h)}\frac{dh}{dE[x_{t+1}]}>0\\\frac{\partial Y_2}{\partial E[x_{t+1}]}&=-\frac{\alpha Y_2}{(1-\alpha)(1-h)}\frac{dh}{dE[x_{t+1}]}>0\end{aligned}$$

Using the fact that expected surplus is an increasing function of the amount of loan $\mathbb{E}[S_2|h] = (1-h)Y_2 - K_2 = \frac{1-\alpha^2}{\alpha^2}K_2$:

$$\frac{d\mathbb{E}[S|h]}{dE[x_{t+1}]} = \frac{1-\alpha^2}{\alpha^2}\frac{dK_2}{dE[x_{t+1}]} > 0$$

When $E < ((1-h)^{\frac{1}{1-\alpha}} - r_1)$, the probability of bank failure falls as expectations improve. When $E > ((1-h)^{\frac{1}{1-\alpha}} - r_1)$ the probability of bank failure is 0. This happens over a larger set of values for E as the default hazard falls. Overall, the probability of bank failure is non-increasing as expectations about the macro state improve.

In the constrained case:

$$\begin{aligned}
\frac{\partial K_2}{\partial E[x_{t+1}]} = -\frac{E(1-\xi)}{(h+\xi(1-h))^2}\frac{dh}{dE[x_{t+1}]} &= -\frac{(1-\xi)K_2}{h+\xi(1-h)}\frac{dh}{dE[x_{t+1}]} > 0 \\
\frac{\partial Y_2}{\partial E[x_{t+1}]} &= -\frac{\alpha(1-\xi)Y_2}{h+\xi(1-h)}\frac{dh}{dE[x_{t+1}]} > 0
\end{aligned}$$

Not that we can express the link between output and investment in the following way:

$$\begin{aligned}
Y_2 &= A\left(\frac{1+\lambda^{FL}(\xi+(1-\xi)h)}{\alpha^2 A(1-h)}\right)^{\frac{\alpha}{1-\alpha}} \\
&= A\left(\frac{1+\lambda^{FL}(\xi+(1-\xi)h)}{\alpha^2 A(1-h)}\right)\left(\frac{1+\lambda^{FL}(\xi+(1-\xi)h)}{(1-h)\alpha}\right)^{\frac{1}{1-\alpha}} \\
&= \frac{1+\lambda^{FL}(\xi+(1-\xi)h)}{\alpha^2(1-h)}K_2 \\
S &= (1-h)Y_2 - K_2 \\
&= \frac{1+\lambda^{FL}(\xi+(1-\xi)h)-\alpha^2}{\alpha^2}K_2 \\
\frac{dS}{dh} &= \frac{\lambda^{FL}(1-\xi)}{\alpha^2}K_2 + \frac{1+\lambda^{FL}(\xi+(1-\xi)h)-\alpha^2}{\alpha^2}\frac{dK_2}{dh} \\
&= \frac{\lambda^{FL}(1-\xi)}{\alpha^2}K_2 - \frac{1+\lambda^{FL}(\xi+(1-\xi)h)-\alpha^2}{\alpha^2}\frac{1-\xi}{\xi+(1-\xi)h}K_2 \\
&= \frac{K_2(1-\xi)}{\alpha^2}\left(\lambda^{FL} - \frac{1+\lambda^{FL}(\xi+(1-\xi)h)-\alpha^2}{\xi+(1-\xi)h}\right) \\
&= \frac{K_2(1-\xi)}{\alpha^2}\left(\frac{\lambda^{FL}(\xi+(1-\xi)h) - 1 - \lambda^{FL}(\xi+(1-\xi)h)+\alpha^2}{\xi+(1-\xi)h}\right) \\
&= \frac{K_2(1-\xi)}{\alpha^2}\frac{-1+\alpha^2}{\xi+(1-\xi)h} < 0 \\
\frac{dS}{dE[x_{t+1}]} &= \frac{dS}{dh}\frac{dh}{dE[x_{t+1}]} > 0
\end{aligned}$$